

Quiz #8

Monday, December 4 2017

Duration: 20 min

NAME: _____

Please write clearly and properly. Justify your answers carefully.

Problem	Grade
1	
2	
Total	

Problem 1 (~ 5 points).

Consider the ring $R = \mathbb{Z}/5\mathbb{Z}$.

(1) Complete the table of addition in R :

+	[0]	[1]	[2]	[3]	[4]
[0]					
[1]					
[2]					
[3]					
[4]					

(2) Complete the table of multiplication in R :

·	[0]	[1]	[2]	[3]	[4]
[0]					
[1]					
[2]					
[3]					
[4]					

- (3) Show that every element of $R - \{[0]\}$ admits a multiplicative inverse. *Hint: use the previous question.* What kind of ring is R ? Explain your answer.

Problem 2 (~ 5 points).

Consider the set $C \subset \mathcal{M}_2(\mathbb{R})$ consisting of matrices M of the form:

$$M = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

where a and b are two real numbers.

- (1) Carefully prove that $(C, +, \cdot)$ is a ring. *Note: Here $+$ and \cdot denote the usual addition and multiplication of matrices.*

(2) Carefully prove that the function

$$\varphi: \mathbb{C} \rightarrow \mathbb{C}$$
$$z = a + ib \mapsto \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

is an isomorphism of rings.

(3) *Bonus question.* Consider the matrix $M = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$. Compute M^{100} . *Hint: use the previous question.*