

Quiz #6

Monday, November 6 2017

Duration: 20 min

NAME: _____

Please write clearly and properly. Justify your answers carefully.

| Problem | Grade |
|----------------|--------------|
| 1 | |
| 2 | |
| Total | |

Problem 1 (~ 7 points).

Consider the following element of the group S_5 (the symmetric group on 5 letters):

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$$

(1) Compute $\sigma^2, \sigma^3, \dots, \sigma^6$.

(2) What is the order of σ in S_5 ?


We recall that the order of an element x in a group is the smallest positive integer n such that x^n is the identity element.

(3) Is σ a generator of S_5 ?

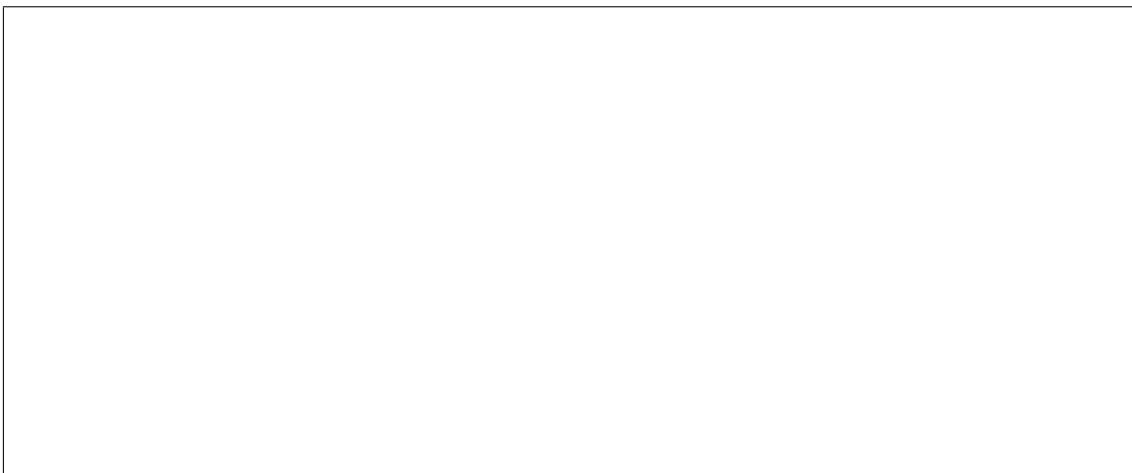
(4) What are the orbits of σ ?

(5) Write σ as a product of disjoint cycles.

(6) Write σ as a product of transpositions. *Hint: check that $(i, j, k) = (i, k)(i, j)$.*



(7) What is the signature of σ ?



Problem 2 (~ 2 points).

Show that the map:

$$\begin{aligned} \text{sign}: S_n &\rightarrow \{-1, 1\} \\ \sigma &\mapsto \text{sign}(\sigma) \end{aligned}$$

is a group homomorphism from the symmetric group S_n to the group $(\{-1, 1\}, \times)$.