

Quiz #5

Monday, October 30 2017

Duration: 40 min

NAME: _____

Please write clearly and properly. Justify your answers carefully.

Problem	Grade
1	
2	
Total	

Problem 1 (~ 8 points).

Consider the group $G = (U_8, \times)$, where U_8 denotes the set of 8th roots of unity in \mathbb{C} .

(1) List all the elements of G in polar form.

Check that $G = \{\zeta^0, \zeta^1, \zeta^2, \zeta^3, \zeta^4, \zeta^5, \zeta^6, \zeta^7\}$ where $\zeta = e^{i\frac{\pi}{4}}$.

(2) Is G a cyclic group? Explain.

(3) Find the subgroup generated by each element of G . Explain.

Hint: Proceed one by one: first find the subgroup generated by ζ^0 , then find the subgroup generated by ζ^1 , etc. Here are the answers that you should find:

$$\langle \zeta^0 \rangle = \{ \zeta^0 \}$$

$$\langle \zeta^1 \rangle = G$$

$$\langle \zeta^2 \rangle = \{ \zeta^0, \zeta^2, \zeta^4, \zeta^6 \}$$

$$\langle \zeta^3 \rangle = G$$

$$\langle \zeta^4 \rangle = \{ \zeta^0, \zeta^4 \}$$

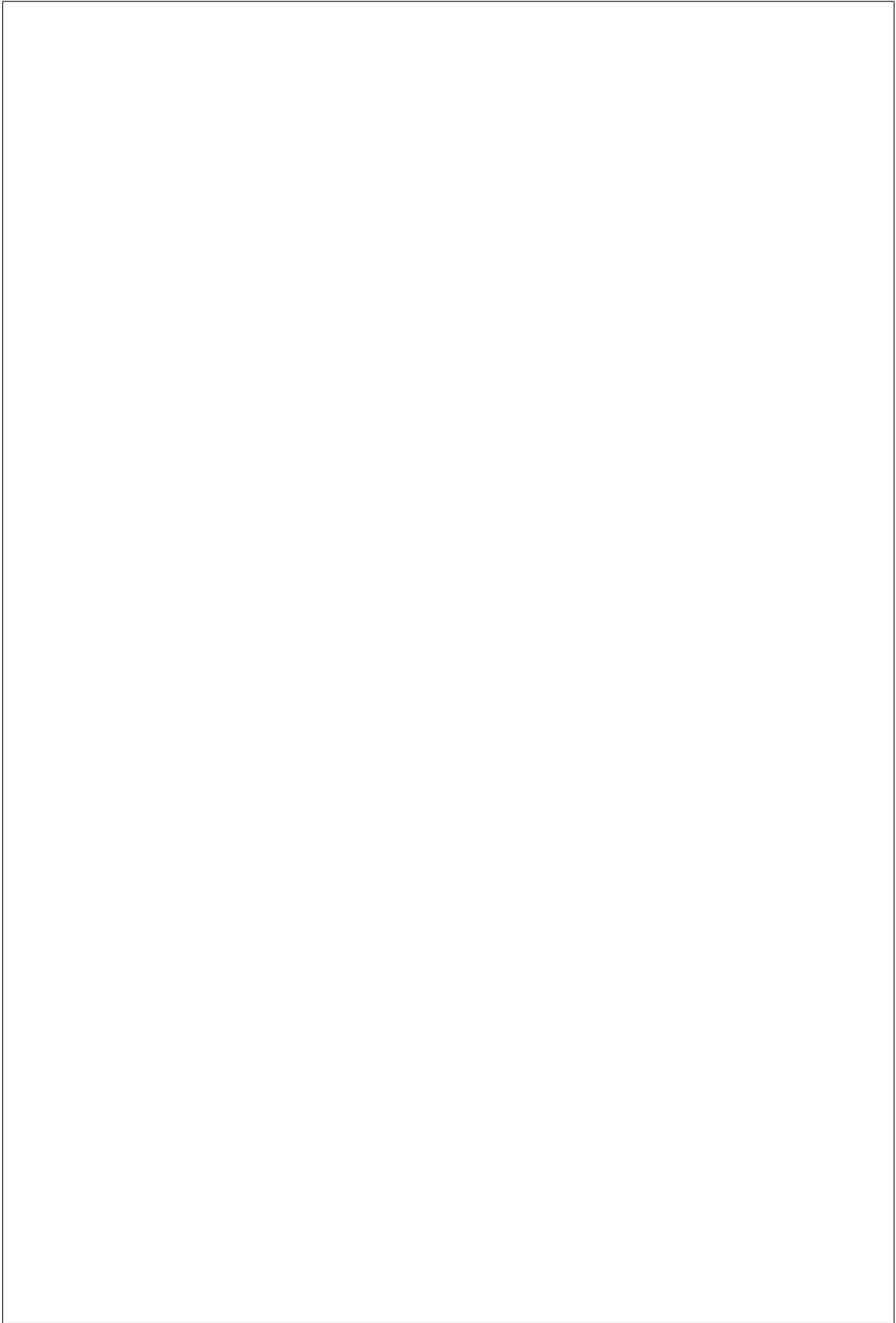
$$\langle \zeta^5 \rangle = G$$

$$\langle \zeta^6 \rangle = \{ \zeta^0, \zeta^2, \zeta^4, \zeta^6 \}$$

$$\langle \zeta^7 \rangle = G .$$



You may continue to write your answer on the next page.



(4) Find all the generators of G . Explain.

We recall that a generator of a cyclic group G is an element $a \in G$ such that $G = \langle a \rangle$.

(5) Find all the subgroups of G . Explain.

Problem 2 (~ 5 points).

(1) State the *Theorem of Euclidean division* in \mathbb{Z} .

(2) State the definition of *greatest common divisor* and *least common multiple* in \mathbb{Z} .
State the definition that we saw in class.

(3) What is the Euclidean division of 29 by 4? *Just write the result, no explanations required.*

(4) What is $10\mathbb{Z} \cap 6\mathbb{Z}$? *Just write the result, no explanations required.*

(5) What is the subgroup generated by 6 and 9 in \mathbb{Z} ? *Just write the result, no explanations required.*