

## Quiz #1

Monday, September 18 2017

**Duration: 20 min**

**NAME:** \_\_\_\_\_

**Please write clearly and properly. Justify your answers carefully.**

<b>Problem</b>	<b>Grade</b>
<b>1</b>	
<b>2</b>	
<b>3</b>	
<b>Total</b>	

**Problem 1** (~ 6 points.).

For any integer  $m$ , let us define the set  $A_m = \{n \in \mathbb{Z} \mid n \text{ is divisible by } m\}$ .

(1) What is  $A_2 \cap A_3$ ?

(2) What is  $\bigcap_{m=2}^{m=+\infty} A_m$ ?

(3) What is  $\bigcup_{m=2}^{m=+\infty} A_m$ ?

**Problem 2** (~ 6 points.).

Consider the function

$$f: \begin{array}{ccc} \mathbb{Z} & \rightarrow & \mathbb{Z} \\ x & \mapsto & 2x + 1 . \end{array}$$

Is  $f$  injective? Is  $f$  surjective? Is  $f$  bijective? Can you change the domain and/or codomain of  $f$ , so that  $f$  becomes bijective?



**Problem 3** (~ 6 points.).

Let  $n$  be some positive integer. Consider the relation  $\mathcal{R}$  on  $\mathbb{Z}$  defined by:

$$a\mathcal{R}b \Leftrightarrow (a - b) \text{ is divisible by } n.$$

- (1) Show that  $\mathcal{R}$  is an equivalence relation.

- (2) Consider the partition of  $\mathbb{Z}$  relative to this equivalence relation. How many subsets does it consist of?