List of topics for the Final Exam

Topics: The list of topics for Exam #1 includes sections 13, 14, 18, 19, 22, 23. However, it is advised to also review previous sections.

Sections 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11

You should review Sections 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 because they contain the foundations for the next sections. At the very least, go through the lecture notes carefully and make sure that you understand and remember everything. Refer to the list of topics for Exam #1 and Exam #2 for details.

Section 13: Group homomorphisms

Group homomorphism. Examples. Properties of group homomorphisms. Image and preimage of subgroups. Kernel. Characterization of injectivity.

Section 14: Factor groups.

Quotient sets. Canonical projection. Normal subgroups. Characterizations of normal subgroups. Quotient groups. Groups $\mathbb{Z}/n\mathbb{Z}$. Fundamental homomorphism theorem. First isomorphism theorem.

Section 18: Rings and fields.

Definition of a ring. Ring vs Rng. Examples of rings. Rings $\mathbb{Z}/n\mathbb{Z}$. Basic properties of rings. Subrings. Characterization of a subring (subring test). Homomorphism of rings. Isomorphism of rings. Invertible elements in a ring. Fields. Examples of fields.

Section 19: Integral domains

Zero divisors. Integral domains. Solving equations in integral domains vs other rings. Zero divisors in $\mathbb{Z}/n\mathbb{Z}$. Characteristic of a ring. Examples.

Section 22: Ring of polynomials

Definition of polynomials over a commutative ring. Ring of polynomials. Degree of a polynomial. Basic properties. Evaluation homomorphism. Polynomial function associated to a polynomial. Zero (or root) of a polynomial.

Section 23: Factorization of polynomials over a field

Euclidean division of polynomials. Characterization of a root. Multiplicity of roots. Divisibility in $F[x]$. Irreducible polynomials. Factorization theorem. Split polynomials and algebraically closed fields.

Review material:

> **Lecture notes** and supporting **textbook**.

> **Homework exercises:** refer to the online course schedule for the homework assignments corresponding to the topics in the list of topics.

> **Quizzes:** Quizzes 7, 8, 9. Refer to the online course schedule for the quizzes and their solutions.

General advice

- Your lecture notes from class should be your primary source of information. You are expected to know all the material in your lecture notes, and no other. Review your lecture notes regularly and thoroughly.

- Remember that all past quizzes, tests and homework exercises are available on the course web page. Make sure you go over all of them (or as many as you can).

- It is of the utmost importance that you work hard on your homework problems: do as many exercises as possible, and as seriously as possible. Just finding the correct answer is not sufficient: you need to understand why it is correct and you need to be able to write the proper justifications.

- I am happy to answer your questions, as long as: 1. They are mathematical questions, and 2. You have made a genuine effort to think about your question before contacting me.

- The best way to prepare for the exams is to work regularly: make sure you understand all the material as it is being taught, do many homework exercises, etc. Don't wait until the last moment to prepare. Don't try to guess what will be on the test, your time is best spent preparing for every possibility.

- Do not expect the test to be solely a direct application of the material you reviewed. It will require you to be capable of original thinking.

- Calculators will not be allowed, nor any other resources.