

## List of topics for Exam #2

**Topics:** The list of topics for Exam #1 includes sections 6, 8, 9, 10, 11. However, it is advised to also review previous sections.

### **Sections 0, 1, 2, 3, 4, 5**

You should review Sections 0, 1, 2, 3, 4, 5, because they contain the foundations for the next sections. At the very least, go through the lecture notes carefully and make sure that you understand everything.

### **Section 6: Cyclic groups**

Subgroup of a group generated by one element: definition, general form. Order of an element of a group. Definition of a cyclic group. Theorem of Euclidean division. Theorem on subgroups of cyclic groups. Subgroups of  $(\mathbb{Z}, +)$ . Definition of the GCD and the LCM in  $\mathbb{Z}$ . Coprime integers. Theorem on the structure of any cyclic group.

### **Section 8: Groups of permutations.**

Group of permutations of any set. Symmetric group on  $n$  letters. Notations and Terminology. Dihedral group. Cayley's theorem.

### **Section 9: Orbits, cycles and the alternating group.**

Orbits of a permutation. Cycles. Theorem on the decomposition of a permutation as a product of disjoint cycles. Transpositions. Theorem on the decomposition of permutations as a product of transpositions. Signature. Discriminant and relation to signature. Alternating group on  $n$  letters.

### **Section 10: Cosets and the theorem of Lagrange.**

Definition of left and right cosets. Examples. Abelian notation. Theorem of Lagrange. Example of application: any group of prime order is cyclic. Index of a subgroup.

### **Section 11: Direct products**

Definition of the direct product of 2 groups. Examples. Generalization to the direct product of  $N$  groups. Abelian notation. Chinese theorem. Theorem on the structure of abelian groups (they decompose as products of cyclic groups).

### **Review material:**

- > **Lecture notes** and supporting **textbook**.
- > **Homework exercises:** refer to the online course schedule for the homework assignments corresponding to the topics in the list of topics.
- > **Quizzes:** Quizzes 5, 6. Refer to the online course schedule for the quizzes and their solutions.

### **General advice**

- Your lecture notes from class should be your primary source of information. You are expected to know all the material in your lecture notes, and no other. Review your lecture notes regularly and thoroughly.
- Remember that all past quizzes, tests and homework exercises are available on the course web page. Make sure you go over all of them (or as many as you can).
- It is of the utmost importance that you work hard on your homework problems: do as many exercises as possible, and as seriously as possible. Just finding the correct answer is not sufficient: you need to understand why it is correct and you need to be able to write the proper justifications.
- I am happy to answer your questions, as long as: 1. They are mathematical questions, and 2. You have made a genuine effort to think about your question before contacting me.
- The best way to prepare for the exams is to work regularly: make sure you understand all the material as it is being taught, do many homework exercises, etc. Don't wait until the last moment to prepare. Don't try to guess what will be on the test, your time is best spent preparing for every possibility.
- Do not expect the test to be solely a direct application of the material you reviewed. It will require you to be capable of original thinking.
- Calculators will not be allowed, nor any other resources.