RUTGERS
UNIVERSITY | NEWARK

21:640:403
Complex variables

Fall 2016

Test #2

Monday, November 14 2016

NAME: ______________________

Please write clearly and properly.
Always justify your answers!

Problem	Grade
1	
2	
3	
4	
5	
Total	

Problem 1 (~5 points)**.** Consider the function

$$\begin{aligned} f\colon \mathbb{C}^* &\to \mathbb{C} \\ z &\mapsto \frac{1}{z}\,. \end{aligned}$$

(1) What kind of function of f? Say all the qualifiers that apply.

(2) Writing z in algebraic form as $z = x + iy$, express $f(z)$ in algebraic form in terms of x and y. Then prove that f is holomorphic using the Cauchy-Riemann equations.

Problem 2 (~4 points)**.** Let n be a positive integer, and consider the polynomial P_n defined by:

$$P_n(z) = 1 + z + \frac{z^2}{2} + \cdots + \frac{z^n}{n!}$$

(1) Find the roots of P_1 and P_2.

(2) Show that for any n, P_n cannot have a root of multiplicity > 1.

Problem 3 (~3 points)**.** Let z be a complex number. Find a formula expressing $\sin(2z)$ in terms of $\sin(z)$ and $\cos(z)$, and prove it.

Problem 4 (~3 points)**.** Let $\mathrm{Log}\colon \Omega \to \mathbb{C}$ denote the principal branch of the complex logarithm. Is it true that $\mathrm{Log}(z_1 z_2) = \mathrm{Log}(z_1) + \mathrm{Log}(z_2)$ for any complex numbers z_1 and z_2 in Ω? *Either prove that it is true or prove that it is false by providing a counter-example.*

Problem 5 (~3 points)**.** Do the following series of complex numbers converge? *Explain your answers carefully.*

(1)

$$\sum_{k\geqslant 0} \frac{e^{i\pi/k}}{k^2}$$

(2)

$$\sum_{k\geqslant 0} \frac{(1+i)^k}{4\,e^{i\cos(k^2)}}$$