

Test #1

Monday, October 10 2016

NAME: _____

Please write clearly and properly. Justify all your answers.

Problem	Grade
1	
2	
3	
Total	

Problem 1 (~4 points). Solve the equation:

$$z^2 + z + \frac{1 - 2i}{4} = 0 .$$

Problem 2 (~7 points). Consider the complex exponential function:

$$\begin{aligned}\exp: \mathbb{C} &\rightarrow \mathbb{C} \\ z &\mapsto e^z\end{aligned}$$

- (1) What is the domain of definition of the function \exp ? What is its target?

- (2) Consider the complex numbers $z_1 = 0$, $z_2 = i\pi$, $z_3 = 5i\pi$ and $z_4 = 3 - i\pi/2$. Compute the image of each of these complex numbers by the function \exp . Write your answers in algebraic form.

- (3) Consider the complex numbers $y_1 = 1$, $y_2 = 0$, $y_3 = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ and $y_4 = 2i$. Find all the preimages of each of these complex numbers by the function \exp . Write your answers in algebraic form.



- (4) Is the function \exp injective? Is it surjective? Is it bijective? Justify all your answers.

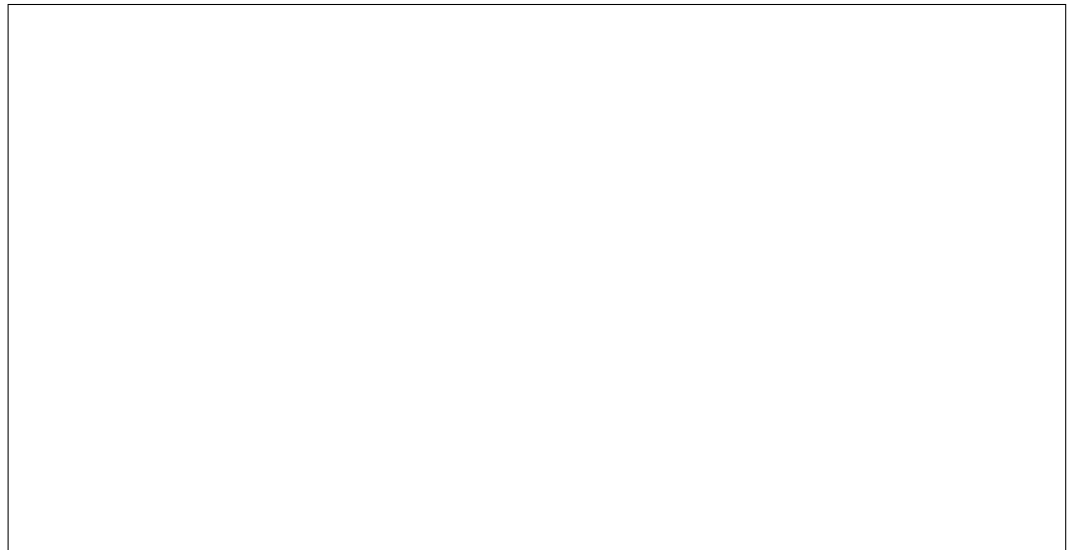


Problem 3 (~13 points).

- (1) (i) Let x be a real number. Define the set $V_x \subseteq \mathbb{C}$ as follows:

$$V_x = \{z \in \mathbb{C} : \operatorname{Re}(z) = x\} .$$

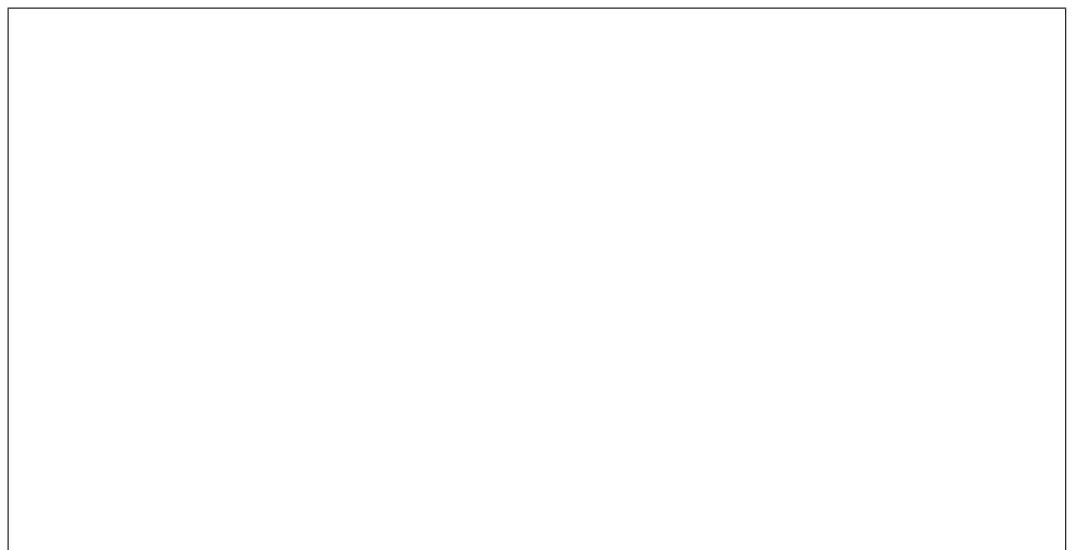
Sketch V_x in the complex plane for $x = 2$.



- (ii) Let y be a real number. Define the set $H_y \subseteq \mathbb{C}$ as follows:

$$H_y = \{z \in \mathbb{C} : \operatorname{Im}(z) = y\} .$$

Sketch H_y in the complex plane for $y = 1$.



- (iii) Let x_1 and x_2 be two real numbers. Define the set $V_{x_1, x_2} \subseteq \mathbb{C}$ as follows:

$$V_{x_1, x_2} = \{z \in \mathbb{C} : x_1 \leq \operatorname{Re}(z) \leq x_2\}.$$

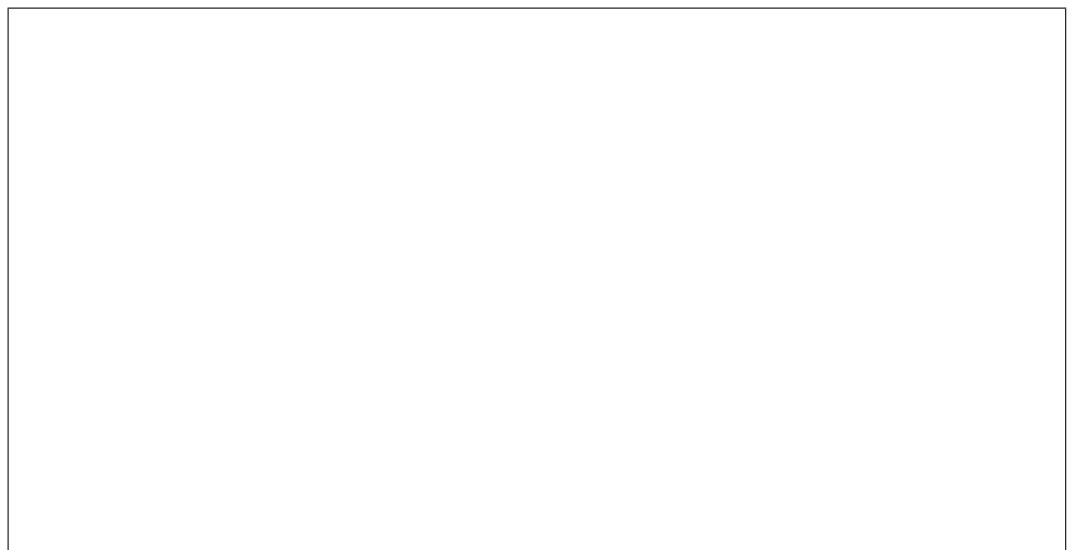
Sketch V_{x_1, x_2} in the complex plane for $(x_1, x_2) = (-1, 2)$.



- (iv) Let y_1 and y_2 be two real numbers. Define the set $H_{y_1, y_2} \subseteq \mathbb{C}$ as follows:

$$H_{y_1, y_2} = \{z \in \mathbb{C} : y_1 \leq \operatorname{Im}(z) \leq y_2\}.$$

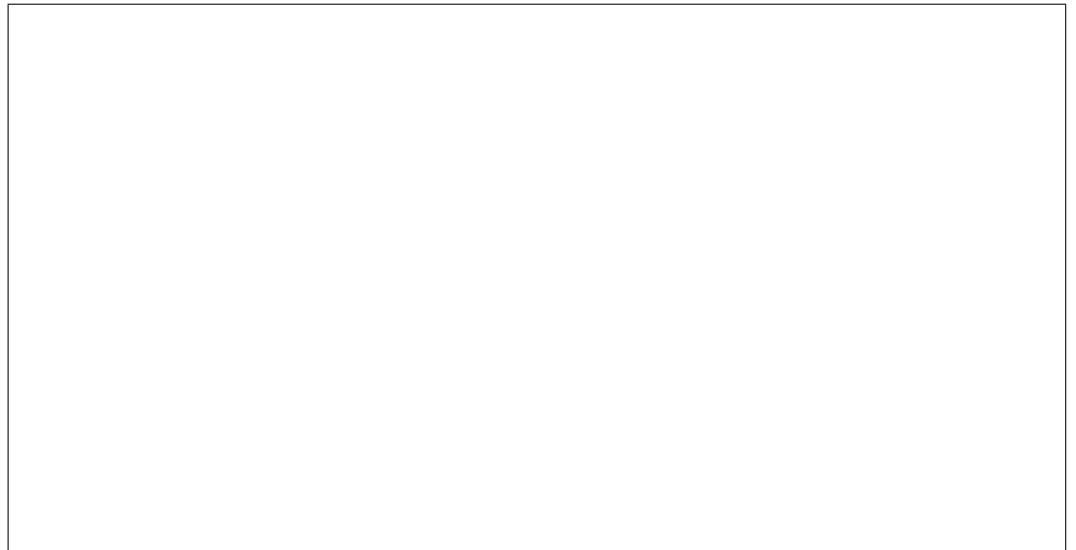
Sketch H_{y_1, y_2} in the complex plane for $(y_1, y_2) = (1, 3)$.



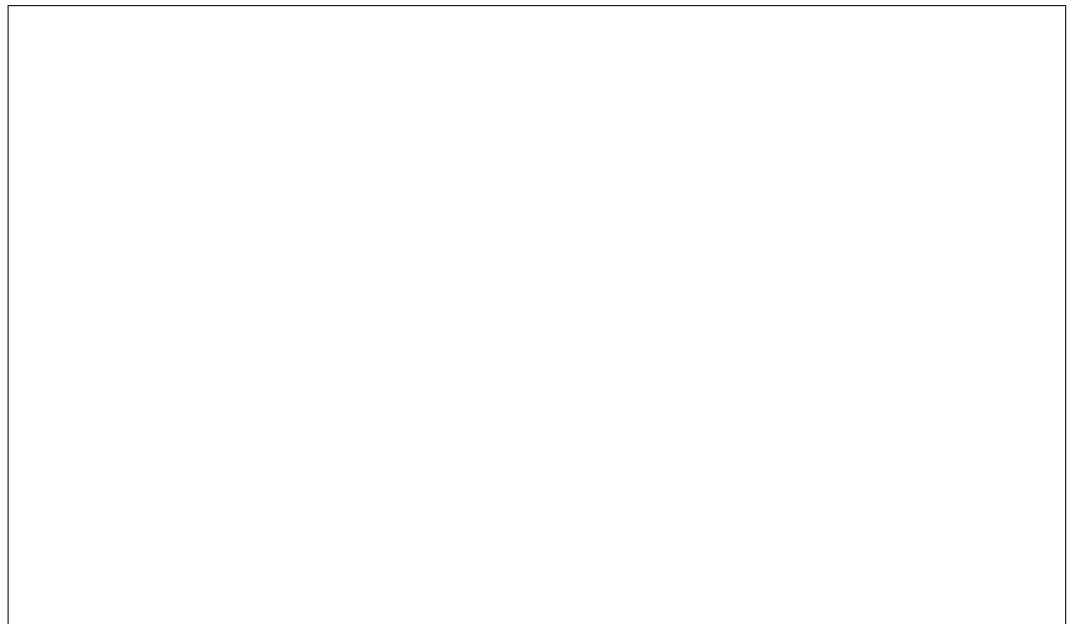
- (v) Let x_1, x_2, y_1, y_2 be four real numbers. Define the set $R_{x_1, x_2, y_1, y_2} \subseteq \mathbb{C}$ as follows:

$$R_{x_1, x_2, y_1, y_2} = V_{x_1, x_2} \cap H_{y_1, y_2} .$$

Sketch R_{x_1, x_2, y_1, y_2} in the complex plane for $(x_1, x_2, y_1, y_2) = (-1, 2, 1, 3)$.



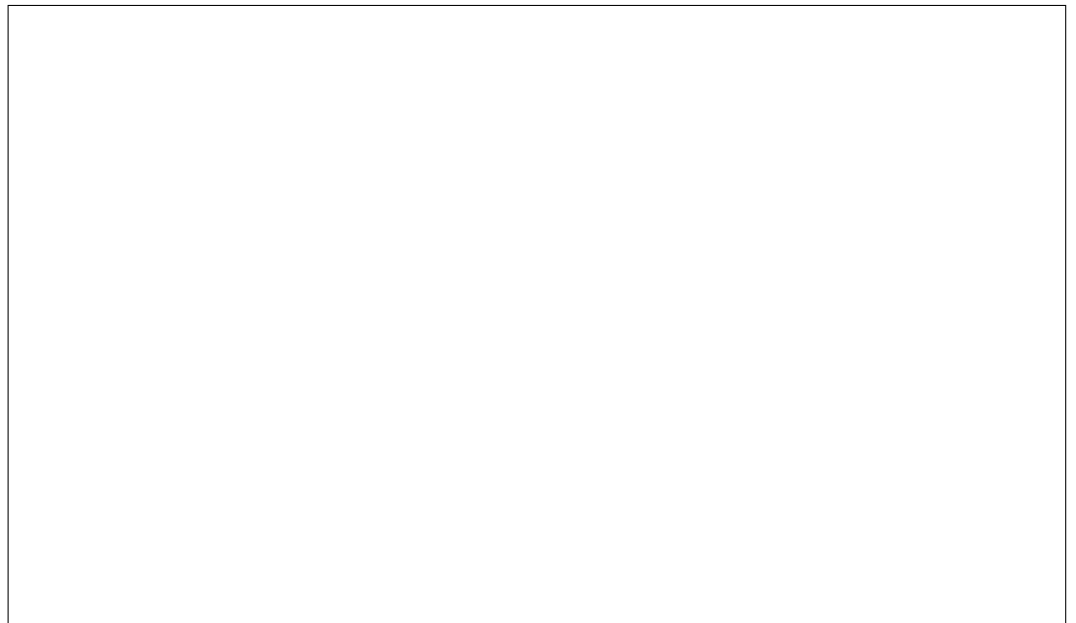
- (2) (i) Is V_{x_1, x_2} open? Is it closed? Is it compact? Is it connected?



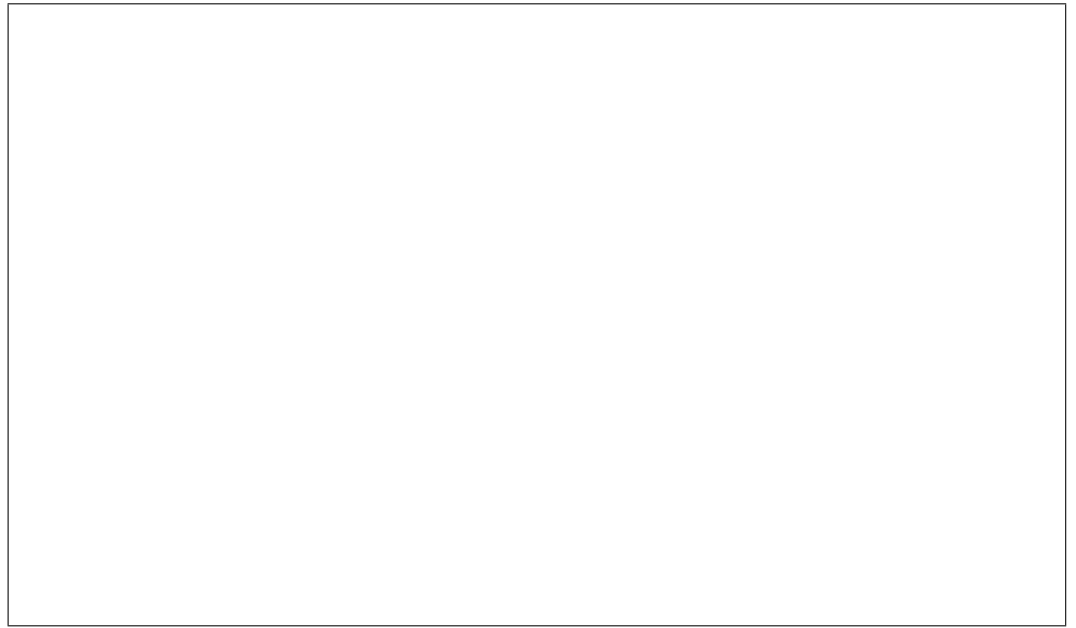
- (ii) Is R_{x_1, x_2, y_1, y_2} convex? Is it star-shaped? Is it simply connected? Is it path-connected?



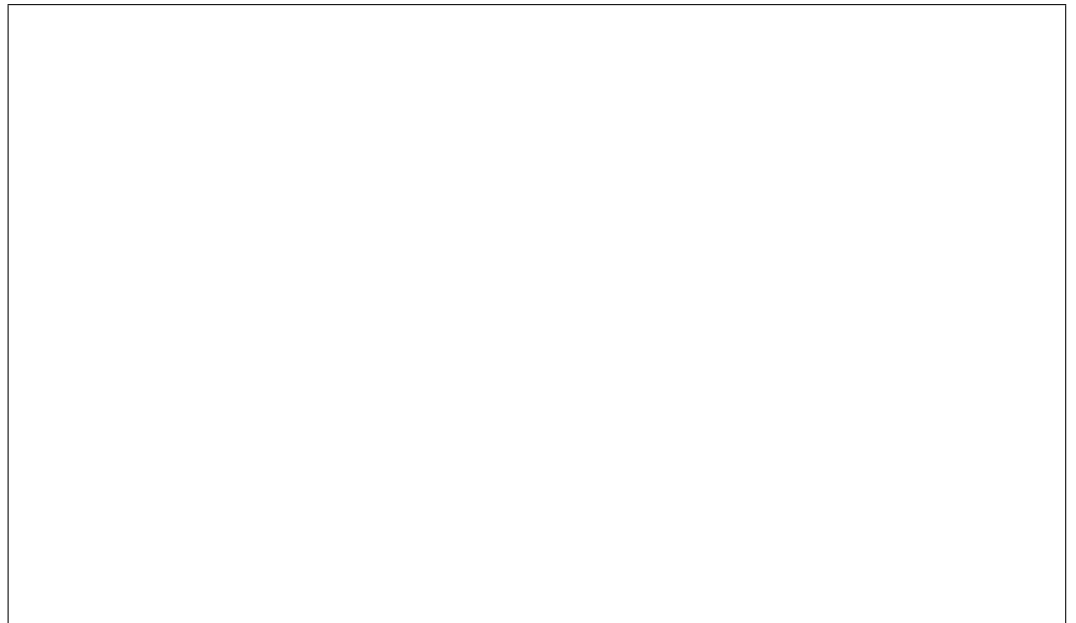
- (3) (i) What is the image of V_x by the exponential function? Sketch $\exp(V_x)$ in the complex plane for some value of x of your choosing.



- (ii) What is the image of H_y by the exponential function? Sketch $\exp(H_y)$ in the complex plane for some value of y of your choosing.



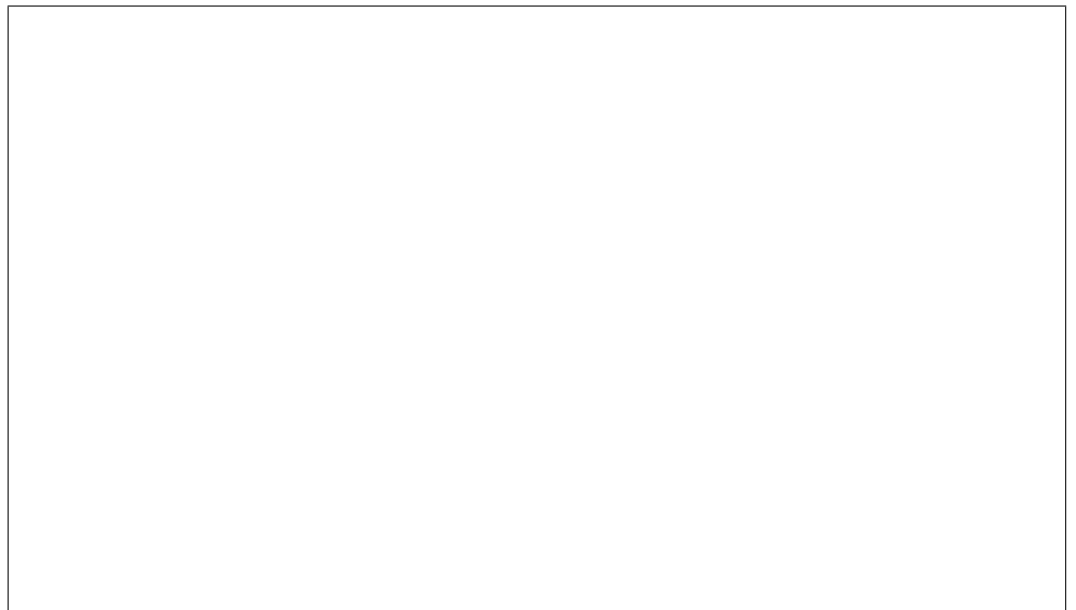
- (iii) What is the image of V_{x_1, x_2} by the exponential function? Sketch $\exp(V_{x_1, x_2})$ in the complex plane for some values of x_1 and x_2 of your choosing.



- (iv) What is the image of H_{y_1, y_2} by the exponential function? Sketch $\exp(H_{y_1, y_2})$ in the complex plane for some values of y_1 and y_2 of your choosing.



- (v) What is the image of R_{x_1, x_2, y_1, y_2} by the exponential function? Sketch $\exp(R_{x_1, x_2, y_1, y_2})$ in the complex plane for some values of x_1 , x_2 , y_1 and y_2 of your choosing.



(4) (i) Is $\exp(R_{x_1, x_2, y_1, y_2})$ open? Is it closed? Is it compact? Is it connected?



(ii) Is $\exp(R_{x_1, x_2, y_1, y_2})$ convex? Is it star-shaped? Is it simply connected?
Is it path-connected?

