

## Quiz #4

Monday, October 17 2016

**NAME:** \_\_\_\_\_

**Please write clearly and properly.**

<b>Problem</b>	<b>Grade</b>
<b>1</b>	
<b>2</b>	
<b>Total</b>	

**Problem 1** (~ 4 points.).

True or false? *No explanation is required.*

- (1) The image of an open set by a continuous function is open.
- (2) The image of a closed set by a continuous function is closed.
- (3) The image of a compact set by a continuous function is compact.
- (4) The image of a bounded set by a continuous function is bounded.
- (5) The image of a connected set by a continuous function is connected.
- (6) The image of a convex set by a continuous function is convex.
- (7) The image of a simply connected set by a continuous function is simply connected.
- (8) The image of a finite set by a continuous function is finite.

*Hint: Counter-examples to some of the statements above are provided by  $f(A)$  where:*

- *$f$  is the exponential function and  $A = \mathbb{C}$ .*
- *$f$  is the function  $z \mapsto 1/z$  and  $A = D^*(0, 1) = D(0, 1) \setminus \{0\}$ .*
- *$f$  is the function  $z \mapsto 1/(1 + |z|)$  and  $A = \mathbb{C}$ .*

**Problem 2** (~ 4 points.).

True or false? *No explanation is required.*

- (1) The preimage of an open set by a continuous function is open.
- (2) The preimage of a closed set by a continuous function is closed.
- (3) The preimage of a compact set by a continuous function is compact.
- (4) The preimage of a bounded set by a continuous function is bounded.
- (5) The preimage of a connected set by a continuous function is connected.
- (6) The preimage of a convex set by a continuous function is convex.
- (7) The preimage of a simply connected set by a continuous function is simply connected.
- (8) The preimage of a finite set by a continuous function is finite.

*Hint: Counter-examples to some of the statements above are provided by  $f^{-1}(A)$  where  $f: \mathbb{C} \rightarrow \mathbb{C}$  is the exponential function and  $A = \{1\}$ .*