RUTGERS UNIVERSITY | NEWARK

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Complex variables

Fall 2016

Homework exercises #7

Problem 1. Find the Taylor expansion of the polynomial P at $z = a$, where:

(1) $P(z) = z^2 - 2z - 3$ and $a = 3$.
(2) $P(z) = z^5$ and $a = 0$.
(3) $P(z) = z^5$ and $a = 1$.
(4) $P(z) = (z + 1)^4 + z$ and $a = -1$.
(5) $P(z) = z^3 - 2iz + i$ and $a = 1 - i$.

Problem 2. Without using the fundamental theorem of algebra, prove that a polynomial of degree n cannot have a root of order $> n$.

Problem 3. Consider two polynomials P and Q with complex coefficients such that:

$$\begin{aligned} P(z_0) &= Q(z_0) \\ P'(z_0) &= Q'(z_0) \\ &\dots \\ P^{(n)}(z_0) &= Q^{(n)}(z_0) \end{aligned}$$

where z_0 is some complex number and n is some nonnegative integer such that both P and Q have degree at most n. Prove that $P = Q$.

Problem 4. Let n be a nonnegative integer, and consider the polynomial P defined by:

$$P(z) = 1 + z + \frac{z^2}{2} + \cdots + \frac{z^n}{n!}$$

Show that P only has simple roots. *A simple root is by definition a root of multiplicity 1.*

Problem 5.

(1) Find all the roots and the poles (and their orders) of the following rational fraction:

$$f(z) = \frac{1 - z^2}{1 - z^3}$$

(2) Same question for the following rational fraction:

$$f(z) = \frac{z^5 - z^3}{z^3 + (3i - 1)z^2 - (2 + i)z}$$

Problem 6.

(1) Find the poles α_1, α_2 and α_3 of the rational fraction:

$$f(z) = \frac{1}{z^3 + z} .$$

(2) Find the complex numbers c_1, c_2 and c_3 such that:

$$f(z) = \frac{c_1}{z - \alpha_1} + \frac{c_1}{z - \alpha_2} + \frac{c_1}{z - \alpha_3} .$$

This is an example of what is known as a partial fraction decomposition.

Additional exercises
No additional exercises this time.