

Homework exercises #11

Problem 1.

True or false? Explain.

- (1) There exists a nonconstant holomorphic function $f : U \rightarrow \mathbb{C}$, where $U = \mathbb{C}$, such that $f(n) = 0$ for any positive integer n .
- (2) There exists a nonconstant holomorphic function $f : U \rightarrow \mathbb{C}$, where $U = \mathbb{C}$, such that $f(1/n) = 0$ for any positive integer n .
- (3) There exists a nonconstant holomorphic function $f : U \rightarrow \mathbb{C}$, where $U = \mathbb{C}^*$, such that $f(1/n) = 0$ for any positive integer n .
- (4) There exists two different holomorphic functions $f : U \rightarrow \mathbb{C}$ and $g : U \rightarrow \mathbb{C}$, where $U = \mathbb{C}$, such that $f(z) = g(z)$ for all z such that $\text{Im}(z) > 0$.
- (5) There exists two different holomorphic functions $f : U \rightarrow \mathbb{C}$ and $g : U \rightarrow \mathbb{C}$, where $U = \{z \in \mathbb{C} : \text{Im}(z) \neq 0\}$, such that $f(z) = g(z)$ for all z such that $\text{Im}(z) > 0$.

Additional exercises

Here are additional exercises from the [textbook](#):

Exercise 4.1 - 4.8

Exercise 4.11

Exercise 4.16

Exercise 4.27