RUTGERS UNIVERSITY | NEWARK

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Complex variables

Fall 2016

Homework exercises #10

Problem 1.
Find the radius of convergence and the domain of convergence of each of the following power series.

(1)
$$\sum_{n\geqslant 0} z^n$$

(2)
$$\sum_{n\geqslant 0} \frac{(z+1)^n}{n^2}$$

(3)
$$\sum_{n\geqslant 1} \frac{4in\,(z-i)^n}{(n+2)!}$$

(4)
$$\sum_{n\geqslant 0} e^{i\pi/n}\left(\frac{1+i}{2}\right)^n (z-4+i)^n$$

(5)
$$\sum_{n\geqslant 0} \frac{n!\,(z-1)^n}{3^n}$$

Problem 2.
Give an example of a power series that:

(1) converges nowhere on the boundary of its domain of convergence.

(2) converges at every point on the boundary of its domain of convergence.

(3) converges at some point(s), but not all, on the boundary of its domain of convergence.

Problem 3.
Show that any analytic function is locally the uniform limit of a sequence of polynomials.

Problem 4.
Justify that each of the following functions f is analytic at $z_0 = 0$, then find its power series representation centered at z_0 and the radius of convergence of that power series.

(1)
$$f(z) = e^z$$

(2)
$$f(z) = e^{-z}$$

(3)
$$f(z) = e^{iz}$$

(4)
$$f(z) = e^{-iz}$$

(5)
$$f(z) = \cos(z)$$

(6)
$$f(z) = \sin(z)$$

(7)
$$f(z) = \frac{1}{1-z}$$

(8)
$$f(z) = \mathrm{Log}(1+z)$$

Problem 5.
Consider an analytic function $f\colon \mathbb{C}^* \to \mathbb{C}$ whose domain of definition is $\mathbb{C}^*$. Can you predict the radius of convergence of its power series representation at $z_0 = -1$? Verify your prediction for the function $f(z) = \frac{1}{z}$.

Additional exercises
If you have extra time, now or when you come back to this sheet of exercises in the future, work on the following exercises from the textbook:
Exercise 7.34
Exercise 7.35
Exercise 7.26
Exercise 7.27