RUTGERS
UNIVERSITY | NEWARK

21:640:238
Foundations of Modern Math

Summer 2016

Test #2

Thursday, June 23 2016

NAME: _____

Please write clearly and properly.

Problem	Grade
1	
2	
3	
4	
5	
6	
Total	

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Problem 1. Prove that: $\forall n \in \mathbb{Z}$ n is odd if and only if n^3 is odd.

Problem 2. Consider the statement P: "For any integers d, a and b, if d divides a or d divides b, then d divides the product ab".

(1) Rewrite the statement P using mathematical symbols only.

(2) Prove the statement P.

Problem 3. Prove: For any two sets A and B, $A \cup B = A \cap B$ if and only if $A = B$.

Problem 4. Consider the statement: For any integers a_1, a_2, ..., a_N, if the product $a_1 a_2 \ldots a_N$ is odd, then a_1, a_2, ..., a_N are all odd.

(1) Rewrite this theorem using only mathematical symbols.

(2) Write a proof of this theorem.

Problem 5. Write a proof by contradiction for each of the following statements:

(1) There is no greatest even integer.

(2) Let x be a nonnegative real number. If $x < y$ for any positive real number y, then $x = 0$.

Problem 6. Use induction to prove that for any positive integer n, the integer $4^n + 5$ is a multiple of 3.

Hint: For the induction step, you may want to observe that $4^{n+1} + 5 = 4\,(4^n + 5) - 15$.