

Test #1

Thursday, June 9 2016

NAME: _____

Please write clearly and properly.

| Problem | Grade |
|----------------|--------------|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| Total | |

Problem 1. For each $r \in \mathbb{R}^+$, let A_r denote the interval $[1/r, +\infty)$ in \mathbb{R} .

(1) What are the intervals $A_1, A_2, A_3, A_{1/2}, A_{1/3}$?

(2) Is it true that $r > r' \Rightarrow A_r \subseteq A_{r'}$? Explain.

(3) Let $S = \{1, 2, 3\}$. Describe $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$.

(4) Let $S = \{1, 1/2, 1/3\}$. Describe $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$.

(5) Let $S = \mathbb{N}$. Describe $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$.

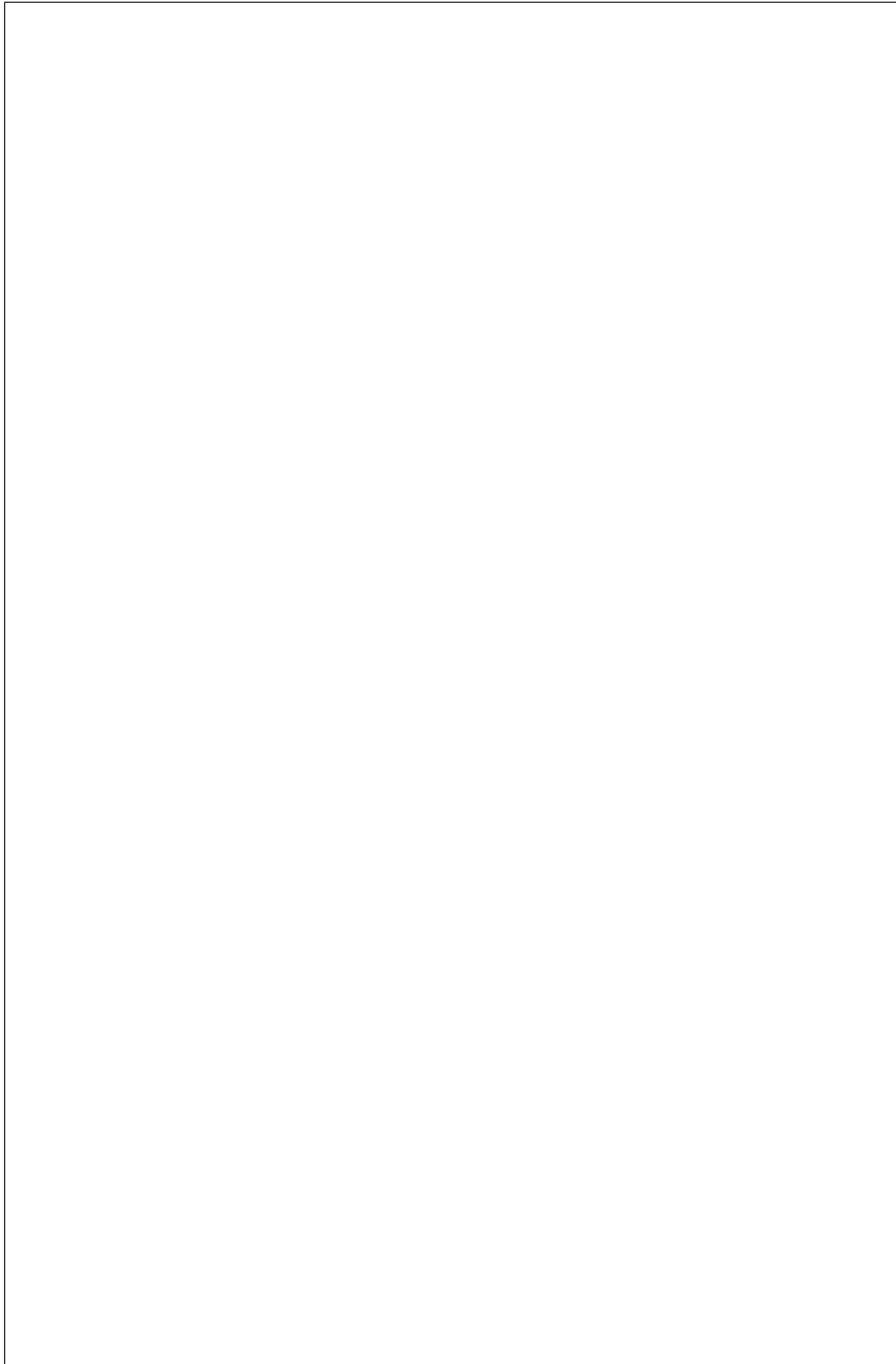
(6) Let $S = \{1/n, n \in \mathbb{N}\}$. Describe $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$.

Problem 2. Give an example of a subset of $\mathbb{R} \times \mathbb{R}$ which is a Cartesian product, and one which is not.

Problem 3. Write the truth table of each of the following statements:

$$P \Rightarrow Q ; Q \Rightarrow P ; (\sim Q) \wedge P ; (\sim P) \vee Q ; (\sim P) \Rightarrow (\sim Q)$$

Which are logically equivalent?



Problem 4. Is P a necessary condition for Q , a sufficient condition for Q , neither, both? *No explanation is required.*

(1) P : Alice has blond hair Q : Alice has hair

(2) P : Bob likes some European countries Q : Bob likes France

(3) P : Pat used to be an athlete Q : The Queen of England is a horse

(4) P : $(R \Rightarrow S)$ Q : $(R \vee S)$

Problem 5. Are the following statements tautologies, contradictions, or neither? Explain briefly.

(1) $P \vee P$

(2) $(\sim P) \Leftrightarrow P$

(3) $P \vee (P \Rightarrow Q)$

(4) $Q \Rightarrow (\sim Q)$

Problem 6. For each of the following open statements $P(x)$ over the domain D , give the set $S \subseteq D$ such that $P(x)$ is true if and only if $x \in S$. *No explanation is required.*

(0) *Example.* $P(x)$: x is prime. $D = \{n \in \mathbb{N} : n < 20\}$.

Answer: $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$.

(1) $P(x)$: $x^2 = 4$. $D = \mathbb{Z}^+$.

Answer: $S =$

(2) $P(x)$: The heir to the English throne is a x . $D = \{\text{horse, mammal, bird}\}$.

Answer: $S =$

(3) $P(x)$: $(1 + x)^2 = 1 + 2x + x^2$. $D = \mathbb{R}$.

Answer: $S =$

(4) $P(x)$: $|x| = 2$. $D = \mathcal{P}(\{\clubsuit, 7, \Omega, \odot\})$.

Answer: $S =$

(5) $P(x)$: $x \subseteq \mathbb{Q}$. $D = \{\mathbb{N}, \mathbb{Z}^-, 2.6\}$.

Answer: $S =$

(6) $P(x)$: $x > 3 \Rightarrow x^2 > 9$. $D = \mathbb{R}$.

Answer: $S =$

(7) $P(x)$: $x \geq 3 \Rightarrow x^2 > 9$. $D = \mathbb{R}$.

Answer: $S =$

(8) $P(x)$: If some birds are x , then the sun is blue. $D = \{\text{pigeons, mammals, white}\}$.

Answer: $S =$