RUTGERS
UNIVERSITY | NEWARK

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Foundations of Modern Math

Summer 2016

Quiz #5

Tuesday, July 5 2016

NAME: ________________________

Please write clearly and properly.

Problem	Grade
1	
2	
3	
Total	

Problem 1. Prove the following statement:

Statement. For any two functions $f : X \to Y$ and $g : Y \to Z$, if f is injective and g is injective, then $g \circ f$ is injective.

Problem 2. Prove the following statement:

Statement. Let $f : X \to Y$ be a function. For any two subsets B and B' of Y, $f^{-1}(B \cap B') = f^{-1}(B) \cap f^{-1}(B')$.

Problem 3. True or false? *No explanation is required.*

(1) Let $A = \{0, 1, 2\}$. There are 8 functions from A to A.

(2) Let

$$\begin{aligned} f\colon \mathbb{Z} &\to \mathbb{Z} \\ x &\mapsto 2x + 1\ . \end{aligned}$$

Then f is neither injective nor surjective.

(3) Let

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^2 + 1\ . \end{aligned}$$

Then $f([0, 1]) = [1, 2]$.

(4) Let f as in (3). Then $f^{-1}([1, 2]) = [0, 1]$.

(5) Let $f : X \to Y$ be a bijective function and let $g = f^{-1}$. Then $f \circ g = g \circ f$.

(6) Let $f : X \to Y$ be a function. Let B and B' be subsets of Y. If $f^{-1}(B) = f^{-1}(B')$, then $B = B'$.

(7) Let $f : X \to Y$, $g : Y \to Z$ and $h : Z \to T$ be three functions. Then $(h \circ g) \circ f = h \circ (g \circ f)$.

(8) Let

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R}^2 \\ x &\mapsto (x, 2x) \end{aligned}$$

and

$$\begin{aligned} g\colon \mathbb{R}^2 &\to \mathbb{R} \\ (x, y) &\mapsto y - x\ . \end{aligned}$$

Then $g \circ f = \mathrm{id}_{\mathbb{R}}$.

(9) Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing function. Then f is injective.

(10) Let $f : X \to X$ be a bijective function. Then $f \circ f$ is well-defined and bijective.