RUTGERS
NEWARK

FOUNDATIONS OF MODERN MATH

21:640:238 (3 credits)

COURSE DESCRIPTION:
Basic concepts on which modern mathematics is founded; language and logical structure of mathematics; elementary set theory, including set operations, relations, and mappings; the structure of the real number system and elements of real analysis. Proof techniques are stressed.

PREREQUISITE:
21:640:136 (Calculus II), or 156 21:640:155 (Honors Calculus I), or permission of instructor.

IMPORTANT NOTE:
Rutgers University requires that all its students complete two writing intensive courses. This course satisfies one of the writing intensive requirements needed for the Mathematics Major.

COURSE OBJECTIVES:
The lower level math courses such as calculus, linear algebra and probability and statistics, cover techniques which can be used in a variety of applications, with an emphasis on calculation. The upper level math courses are of a more abstract nature, and focus on formulating and proving theorems. In particular, they try to answer the question: Why does it all work? This course is a bridge between the lower and upper level courses, and our goal is to learn how to express mathematical ideas precisely and to learn how to understand and write correct mathematical proofs.

TEXTBOOK:
"Doing Mathematics: An Introduction to Proofs and Problem Solving," (2nd edition), by Galovich, published by Cengage.

DEPARTMENT WEB SITE: http://www.ncas.rutgers.edu/math

THIS COURSE COVERS THE FOLLOWING CHAPTERS AND SECTIONS:

Chapter 1: In this chapter we study sets, which, together with functions form the basic building blocks of all mathematical theories. We start by introducing the notions of subset, intersection and union, and then we define partitions of sets (which will later be seen to be equivalent to the notion of an "equivalence relation") as well as Cartesian products of sets.

Chapter 2: We study logical statements, negation, disjunctions and conjunctions of statements. Then we introduce the concepts of implication, tautologies, contradictions and logical equivalence. These structures will allow us to state theorems precisely and, in certain simple cases, we will determine whether a given mathematical statement is true or false.

Chapter 3: We will learn two of the main proof techniques: that of “direct proof” as well as “proof by contradiction”. We will learn how to prove certain statements using a “case by case” analysis.

Chapters 4,5 and 6: We begin to apply the proof techniques to elementary number theory (divisibility and congruences) set theory and we prove some elementary statements about the real number system. We show how to prove the irrationality of various algebraic numbers (such as the square root of two) and we solve logical puzzles using the method of “proof by contradiction”.

Chapter 7. We study relations on sets in general, and equivalence relations in particular. Equivalence relations give us one of the most important way for construction new sets out of old sets. One key example, that of “equivalence modulo an integer n”, is studied in detail.

Chapter 8. We define the all-important concept of “function” as a special kind of relation between two sets. The notions of injective, surjective and bijective are introduced, and are used to give criteria for the existence of an inverse function.

Chapter 9. We learn the technique of “mathematical induction” and apply it to proving various statements from number theory and analysis.

Chapter 12. We apply the proof techniques we’ve learned thus far to develop some of the basic concepts in real analysis: limits of sequences and series are studied, and several of their properties are established. We also give rigorous definitions of “continuity” and “differentiability” and prove some basic results in calculus.

Department of Mathematics & Computer Science
Smith Hall 216, 101 Warren Street, Newark, New Jersey 07102
Phone: (973) 353-5156 Fax: (973) 353-5270