Quiz #9 Solutions

Problem 1.

(1)

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

(2)

$$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

Problem 2.

By the multiplicative principle, there are $2 \times 3 \times 4 = 24$ possible meals. Since $30 > 24$, by the pigeonhole principle, there are at least two people with the same meal.

Problem 3.

(1)

$$a = 2 \times 3 \times 5^2$$
$$B = 2^3 \times 3^2 \times 5$$

(2)

$$\text{GCD}(a, b) = 2 \times 3 \times 5 = 30$$

(3)

$$\text{LCM}(a, b) = 2^3 \times 3^2 \times 5^2 = 1800$$