Quiz #8 Solutions

Problem 1.

(1) There are $2^4 = 16$ such bytes.

(2) There are $2^5 + 2^5 - 2^2 = 60$ such bytes.

(3) There are $C(8, 2) = 28$ such bytes.

(4) There are $C(8, 2) + C(8, 1) + C(8, 0) = 37$ such bytes.

Problem 2.

(1) There are $5^3 = 125$ such words.

(2) There are $5! = 120$ such words.

(3) There are $P(5, 3) = 60$ such words.

(4) There are $C(5, 3) = 10$ subsets of 3 letters.

Problem 3.

By the inclusion-exclusion principle, the answer is $A = M_2 + M_5 - M_{10}$ where:
- M_2 is the number of integers between 20 and 400 that are multiples of 2.
- M_5 is the number of integers between 20 and 400 that are multiples of 5.
- M_{10} is the number of integers between 20 and 400 that are multiples of 2 and multiplies of 5, in other words multiples of 10.

We compute these numbers as follows:
- Since $20 = 2\times10, \ldots, 400 = 2\times200$, M_2 is equal to the number of integers between 10 and 200, that is: $M_2 = 200 - 10 + 1 = 191$.
- Since $20 = 5 \times 4, \ldots, 400 = 5 \times 80$, M_5 is equal to the number of integers between 4 and 80, that is: $M_5 = 80 - 4 + 1 = 77$.
- Since $20 = 10 \times 2, \ldots, 400 = 10 \times 40$, M_{10} is equal to the number of integers between 2 and 40, that is: $M_2 = 40 - 2 + 1 = 39$.

Hence the solution is: $A = 191 + 77 - 39 = 229$.