Quiz #7 Solutions

Problem 1.

(1) Find $Y_0 = 1000$, $Y_1 = 1000 * 1.01 = 1010$, $Y_2 = 1010 * 1.01 = 1020.10$.

(2) Recurrence relation: $Y_n = 1.01 * Y_{n-1}$. Initial condition: $Y_0 = 1000$. This is a geometric sequence.

(3)
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Y(n)
{
    if n==0
    {
        return 1000;
    }
    return 1.01*Y(n-1);
}
```

(4) Solve the recurrence relation: $Y_n = 1.01^n Y_0$. Let us show that this formula result is correct by writing a proof by induction.

Basis step For $n = 0$, the formula is true: $Y_0 = 1.01^0 Y_0$.

Induction step Let $n \in \mathbb{N}_0$ and assume that the formula $Y_n = 1.01^n Y_0$ is true. By the recurrence relation, we know that $Y_{n+1} = 1.01 Y_n$. Since $Y_n = 1.01^n Y_0$ is true, we find that $Y_{n+1} = 1.01^{n+1} Y_0$. Thus the formula is true for $n + 1$. This concludes the proof by induction.

(5) We are looking for the smallest positive integer n such that $Y_n > 2Y_0$. Since $Y_n = 1.01^n Y_0$, we are looking for the smallest positive integer n such that $1.01^n > 2$. Since the natural logarithm lg is an increasing function, this is equivalent to $lg(1.01^n) > lg(2)$. Since $lg(1.01^n) = nlg(1.01)$, we find $n > \frac{lg(2)}{lg(1.01)}$. (Numerically, this is $n = 70$.)