Quiz #6 Solutions

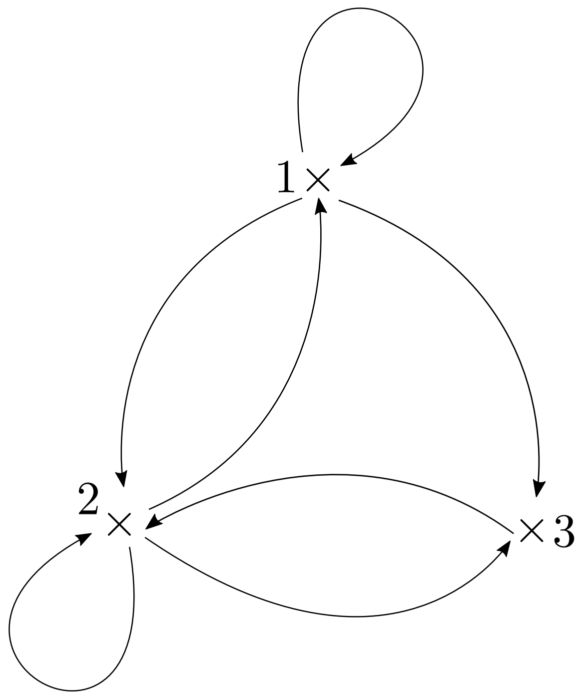
Monday, November 6 2017

Problem 1.

Consider the relation $\mathcal{R}$ on the set $X = \{1, 2, 3\}$ defined by:

$$\mathcal{R} = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,2)\}$$

(1) Here is the digraph of this relation:



(2) This relation is not a function because 1 would have several images (1, 2, and 3), which is not allowed. (Same problem with 2.)

(3) This relation is not reflexive because it is not the case that $3\mathcal{R}3$.

(4) This relation is not symmetric because $1\mathcal{R}3$ but it is not the case that $3\mathcal{R}1$.

(5) This relation is not antisymmetric because $1\mathcal{R}2$ and $2\mathcal{R}1$, even though $1 \neq 2$.

(6) This relation is not transitive because $3\mathcal{R}2$ and $2\mathcal{R}1$, but it is not the case that $3\mathcal{R}1$.

(7) This relation is not a partial order because it is not reflexive (nor antisymmetric, nor transitive).

(8) This relation is not an equivalence relation because it is not reflexive (nor symmetric, nor transitive).

Problem 2 (~ 5 points.)**.**

Consider the relation $\mathcal{R}$ on the set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ defined by:

$$x\mathcal{R}y \;\Leftrightarrow\; x = y \pmod 3 \,.$$

We recall that $x = y$ (mod 3) means, by definition, that 3 divides $(x - y)$.

(1) We need to prove that $\mathcal{R}$ is reflexive, symmetric and transitive:
- $\mathcal{R}$ is reflexive because it is true that for any $x \in X$, $x = x$ (mod 3). Indeed, 3 divides 0.
- $\mathcal{R}$ is symmetric because it is true that for any $x \in X$ and $y \in X$, $x = y$ (mod 3) if and only if $y = x$ (mod 3). Indeed, if 3 divides $x - y$, then 3 divides $y - x$, and conversely.
- $\mathcal{R}$ is transitive because it is true that for any $x \in X$, $y \in X$, and $y \in X$, if $x = y$ (mod 3) and $y = z$ (mod 3), then $x = z$ (mod 3). Indeed, if 3 divides $x-y$ and 3 divides $y-z$, then 3 divides $x-z$ (this is because $x-z = (x-y)+(y-z)$).

(2)

$$\begin{aligned}
[1] &= \{1, 4, 7, 10\}\\
[2] &= \{2, 5, 8\}\\
[3] &= \{3, 6, 9\}\\
[4] &= \{1, 4, 7, 10\}\\
[5] &= \{2, 5, 8\}\\
[6] &= \{3, 6, 9\}\\
[7] &= \{1, 4, 7, 10\}\\
[8] &= \{2, 5, 8\}\\
[9] &= \{3, 6, 9\}\\
[10] &= \{1, 4, 7, 10\}
\end{aligned}$$

(3) The partition of X associated with this equivalence relation is:

$$\{\{1, 4, 7, 10\}, \{2, 5, 8\}, \{3, 6, 9\}\}$$