RUTGERS
UNIVERSITY | NEWARK

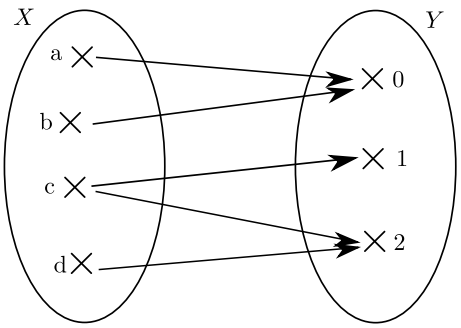
Math 237
Discrete Structures

Fall 2017

Quiz #5 Solutions

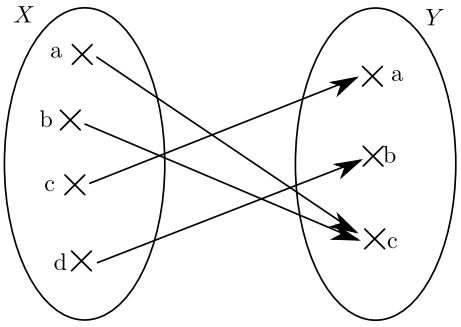
Problem 1.

(1) Here is a diagram representing the relation f:



This relation is not a function because c has several images, which is not allowed for a function.

(2) Here is a diagram representing the relation f:



This relation is a function because every element of the domain has exactly one image in the codomain. It is not injective because a and b have the same image. It is surjective because every element of the codomain has at least one preimage. It is not bijective because it is not injective.

Problem 2.

(1) f is not injective: for instance, $f(0) = f(1/4)$. f is surjective because every $y \in \mathbb{Z}$ has at least one preimage $x \in \mathbb{R}$, for instance $x = y/2$. f is not bijective because it is not injective.

(2) g is injective: if $g(x_1) = g(x_2)$, then $x_1 = x_2$. g is not surjective: for example, $y = 1/3$ has no preimage. g is not bijective because it is not surjective.

(3) The composition $f \circ g$ is well-defined because the codomain of g, namely $\mathbb{R}$, is equal to the domain of f. The composition $f \circ g$ is the function:

$$\begin{aligned} f \circ g \colon \mathbb{Z} &\to \mathbb{R} \\ n &\mapsto n \, . \end{aligned}$$

Indeed, $f(g(n)) = \left\lfloor 2\frac{n}{2} \right\rfloor = n$.

(4) The composition $g \circ f$ is well-defined because the codomain of f, namely $\mathbb{Z}$, is equal to the domain of g. The composition $g \circ f$ is the function:

$$\begin{aligned} g \circ f \colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto \frac{\lfloor 2x \rfloor}{2} \, . \end{aligned}$$

Indeed, $g(f(x)) = \frac{\lfloor 2x \rfloor}{2}$.

Problem 3.

(1) u_n is decreasing and nonincreasing. It is neither increasing nor nondecreasing.

(2) v_n is increasing and nondecreasing. It is neither decreasing nor nonincreasing.

(3) w_n is neither increasing, nor decreasing, nor nonincreasing, nor nondecreasing.

(4) x_n is a constant sequence $x_n = 1$, therefore it is nonincreasing and nondecreasing, and it is neither increasing nor decreasing.

Problem 4.

Here is the list of all substrings of the string b^2a^2c:

$$\begin{aligned} &\lambda \\ &b \\ &a \\ &c \\ &b^2 \\ &ba \\ &a^2 \\ &ac \\ &b^2a \\ &ba^2 \\ &a^2c \\ &b^2a^2 \\ &ba^2c \\ &b^2a^2c \end{aligned}$$