

Quiz #5

Monday, October 30 2017

Duration: 30 min

NAME: _____

Please write clearly and properly.

Problem	Grade
1	
2	
3	
4	
Total	

Problem 1 (~ 6 points.).

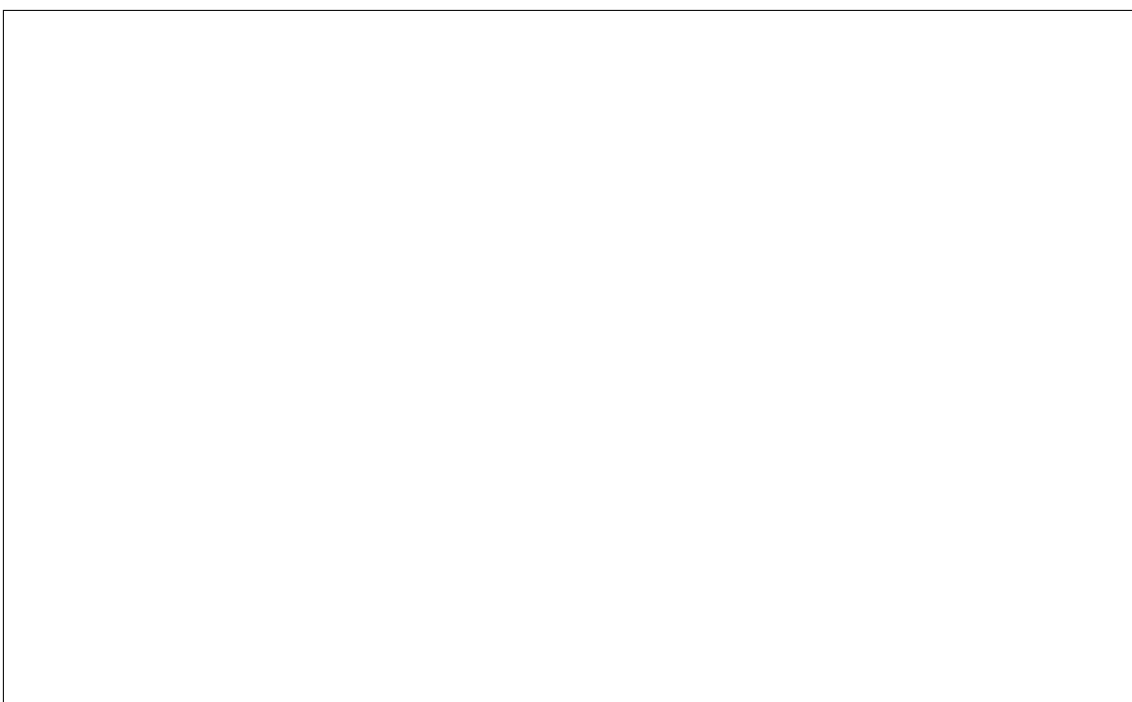
(1) Let $X = \{a, b, c, d\}$ and $Y = \{0, 1, 2\}$. Consider the following relation from X to Y :

$$f = \{(a, 0), (b, 0), (c, 1), (c, 2), (d, 2)\}$$

Draw a diagram representing this relation. Is this relation a function? If yes: is it injective? surjective? bijective? *Explain your answers.*



(2) Same question for $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, and $f = \{(a, c), (b, c), (c, a), (d, b)\}$.



Problem 2 (~ 6 points.).

Consider the following functions:

$$f: \mathbb{R} \rightarrow \mathbb{Z}$$
$$x \mapsto \lfloor 2x \rfloor$$

$$g: \mathbb{Z} \rightarrow \mathbb{R}$$
$$n \mapsto \frac{n}{2}$$

(1) Is f injective? surjective? bijective? Explain.

(2) Is g injective? surjective? bijective? Explain.

(3) Is the composition $f \circ g$ well-defined? If yes, describe the function $f \circ g$. Explain.

(4) Is the composition $g \circ f$ well-defined? If yes, describe the function $g \circ f$. Explain.

Problem 3 (~ 4 points.).

Is the following sequence increasing? decreasing? Nonincreasing? Nondecreasing? *No explanations required.*

(1) $\forall n \in \mathbb{N} \quad u_n = 1 - n.$

(2) $\forall n \in \mathbb{N} \quad v_n = n^2 - n.$

(3) $\forall n \in \mathbb{N} \quad w_n = 1 + (-1)^n.$

(4) $\forall n \in \mathbb{N} \quad x_n = 1 - \sum_{k=-n}^n 2k.$

Problem 4 (~ 2 points.).

Find all substrings of the string b^2a^2c .