

Quiz #3 Solutions

Monday, October 2 2017

Problem 1.

(1) For any sets A and B , $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

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Note: it is understood that all complements are taken relative to a “universe” set U containing A and B .

(2) For any propositions P and Q , $\neg(P \vee Q) = (\neg P) \wedge (\neg Q)$.

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(3) For any propositional function $P(x)$ over some domain D :

$\neg(\forall x \in D \ P(x)) \equiv \exists x \in D \ \neg P(x)$

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Problem 2.

Let C be the set of all countries, and let $P(x, y)$ denote the proposition “ x has a trade agreement with y ” over the domain $C \times C$. The proposition can be written:

$$\exists x \in C \ \exists y \in C \ P(x, y)$$

Note: If we want to eliminate the possibility of a country having a trade agreement with itself, $P(x, y)$ should be “ x has a trade agreement with y and $x \neq y$ ”.

The negation of this proposition is:

$$\forall x \in C \ \forall y \in C \ \neg(P(x, y))$$

which can be stated: “No country has a trade agreement with any other country.”

Problem 3.

(1) True

(2) False

(3) True

(4) True

(5) False

(6) True

(7) True

(8) True