RUTGERS UNIVERSITY | NEWARK

Math 237
Discrete Structures

Fall 2017

Exam #2

Monday, October 13 2017

Duration: 1H20

NAME: ________________________

Please write clearly and properly.

Problem	Grade
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1 (~ 4 points.)**.**

Is the following function injective? Is it surjective? Is it bijective? Explain.

(1)

$$
\begin{aligned}
f_1 : \mathbb{Z} &\to \mathbb{Z} \\
n &\mapsto n^2
\end{aligned}
$$

(2)

$$
\begin{aligned}
f_2 \colon \mathbb{Z} &\to \mathbb{Z} \\
n &\mapsto -4n + 3
\end{aligned}
$$

Problem 2 (~ 4 points.)**.**

Consider the following functions:

$$f\colon \mathbb{R} \longrightarrow \mathbb{R}_{\geqslant 0} \qquad\qquad g\colon \mathbb{R}_{\geqslant 0} \longrightarrow \mathbb{R}$$
$$x \longmapsto x^2 \qquad\qquad x \longmapsto \sqrt{x}$$

(1) Is the composition $f \circ g$ well-defined? If yes, describe the function $f \circ g$. Explain.

(2) Is the composition $g \circ f$ well-defined? If yes, describe the function $g \circ f$. Explain.

Problem 3 (~ 3 points.)**.**

Is the following sequence: increasing? decreasing? nonincreasing? nondecreasing? *Answer all four questions! No explanations required.*

(1)

$$\forall n \in \mathbb{N} \quad u_n = 1 - \left\lfloor \sqrt{1+n} \right\rfloor$$

(2)

$$\forall n \in \mathbb{N} \quad v_n = \frac{(-1)^n}{n}$$

(3)

$$\forall n \in \mathbb{N} \quad w_n = \sum_{k=1}^{n} \frac{(-1)^k}{k}$$

Problem 4 (~ 2 points.)**.**

List all the strings over $X = \{0, 1\}$ of length 3 or less.

Problem 5 (~ 4 points.)**.**

Let n be a positive integer. Denote by $\mathcal{R}$ the relation on $\mathbb{Z}$ defined by:

$$x\mathcal{R}y \iff x = y \pmod{n}$$

We recall that by definition, $x = y \pmod{n}$ means that n is a divisor of $x - y$.

(1) Prove that $\mathcal{R}$ is an equivalence relation.

(2) Assume that $n = 2$. What is the equivalence class of 0? What is the equivalence class of 1? What is the equivalence class of 2? What is the equivalence class of 3?

(3) Going back to the general case of an arbitrary positive integer n, how many equivalence classes are there?

Problem 6 ($\sim$ 6 points.)**.**

Consider the following problem: Given a finite sequence of integers $X = (X(1), X(2), \dots)$ and an integer x, find the sum of all the elements of the sequence that are greater than x.

(1) Write an algorithm (in pseudocode) that solves the problem.

(2) Trace your algorithm for $X = (4, 3, 10, 5)$ and $x = 5$.

(3) What is the complexity of your algorithm? *Carefully explain your answer.*

Problem 7 (~ 4 points.)**.**

True or False? *No explanations required.*

(1) $4n^3 - n^2 + 1 = \mathrm{O}(n^3)$

(2) $4n^3 - n^2 + 1 = \Theta(n^3)$

(3) $4n^3 - n^2 + 1 = \mathrm{o}(n^3)$

(4) $4n^3 - n^2 + 1 = \mathrm{O}(n^4)$

(5) $4n^3 - n^2 + 1 = \mathrm{o}(n^4)$

(6) $n^{1000} + 100n^3 = \mathrm{o}(e^n)$

(7) $n^2 \log n = \mathrm{o}(n^3)$

(8) $n^2 \log n = \Theta(n^2)$