

Exam #2

Monday, October 13 2017

Duration: 1H20

NAME: _____

Please write clearly and properly.

Problem	Grade
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1 (~ 4 points.).

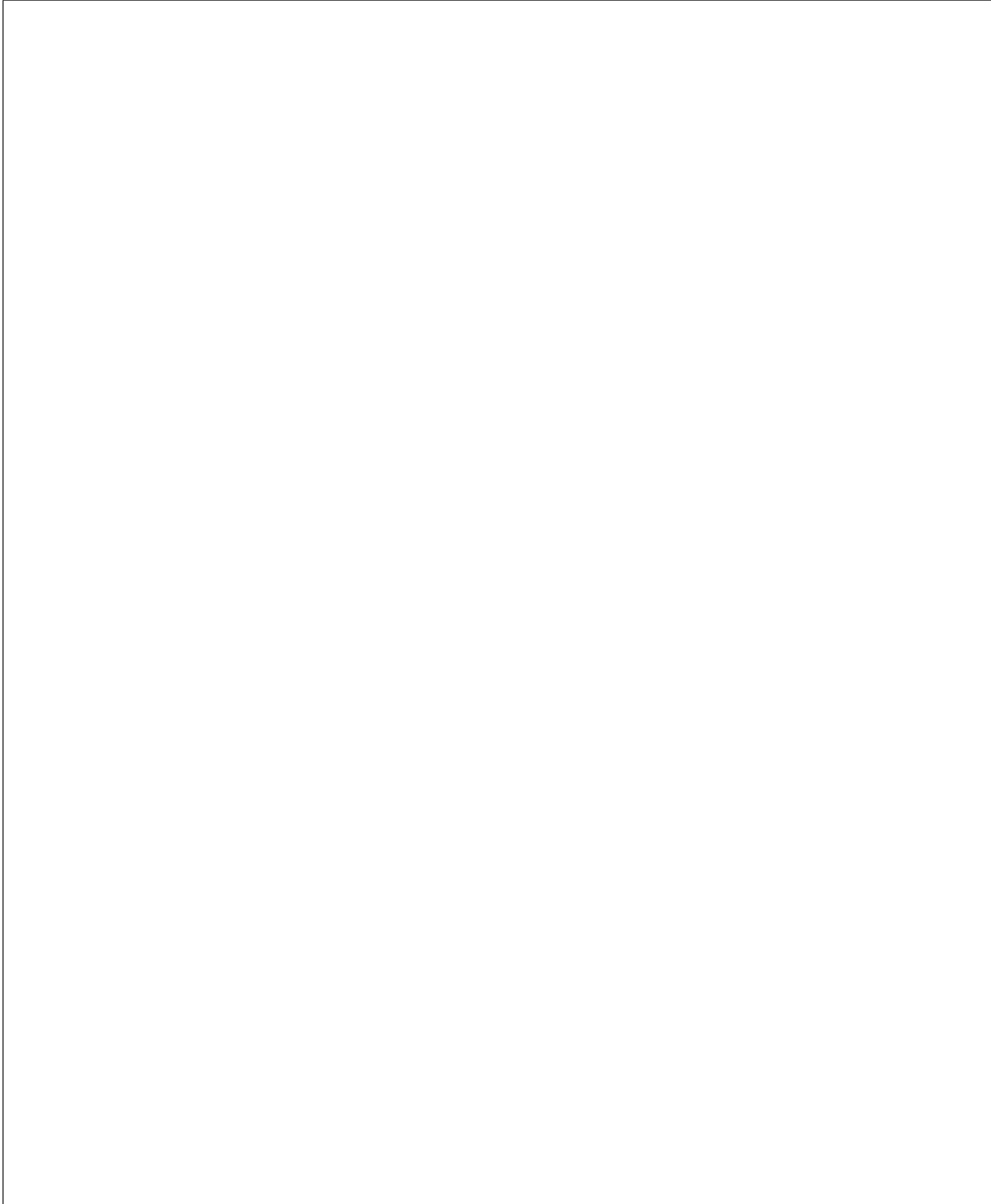
Is the following function injective? Is it surjective? Is it bijective? Explain.

(1)

$$\begin{aligned} f_1: \mathbb{Z} &\rightarrow \mathbb{Z} \\ n &\mapsto n^2 \end{aligned}$$

(2)

$$f_2: \mathbb{Z} \rightarrow \mathbb{Z}$$
$$n \mapsto -4n + 3$$



Problem 2 (~ 4 points.).

Consider the following functions:

$$f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0} \\ x \mapsto x^2$$

$$g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R} \\ x \mapsto \sqrt{x}$$

(1) Is the composition $f \circ g$ well-defined? If yes, describe the function $f \circ g$. Explain.

(2) Is the composition $g \circ f$ well-defined? If yes, describe the function $g \circ f$. Explain.

Problem 3 (~ 3 points.).

Is the following sequence: increasing? decreasing? nonincreasing? nondecreasing?
Answer all four questions! No explanations required.

(1)

$$\forall n \in \mathbb{N} \quad u_n = 1 - \lfloor \sqrt{1+n} \rfloor$$

(2)

$$\forall n \in \mathbb{N} \quad v_n = \frac{(-1)^n}{n}$$

(3)

$$\forall n \in \mathbb{N} \quad w_n = \sum_{k=1}^n \frac{(-1)^k}{k}$$

Problem 4 (~ 2 points.).

List all the strings over $X = \{0, 1\}$ of length 3 or less.



Problem 5 (~ 4 points.).

Let n be a positive integer. Denote by \mathcal{R} the relation on \mathbb{Z} defined by:

$$x\mathcal{R}y \Leftrightarrow x = y \pmod{n}$$

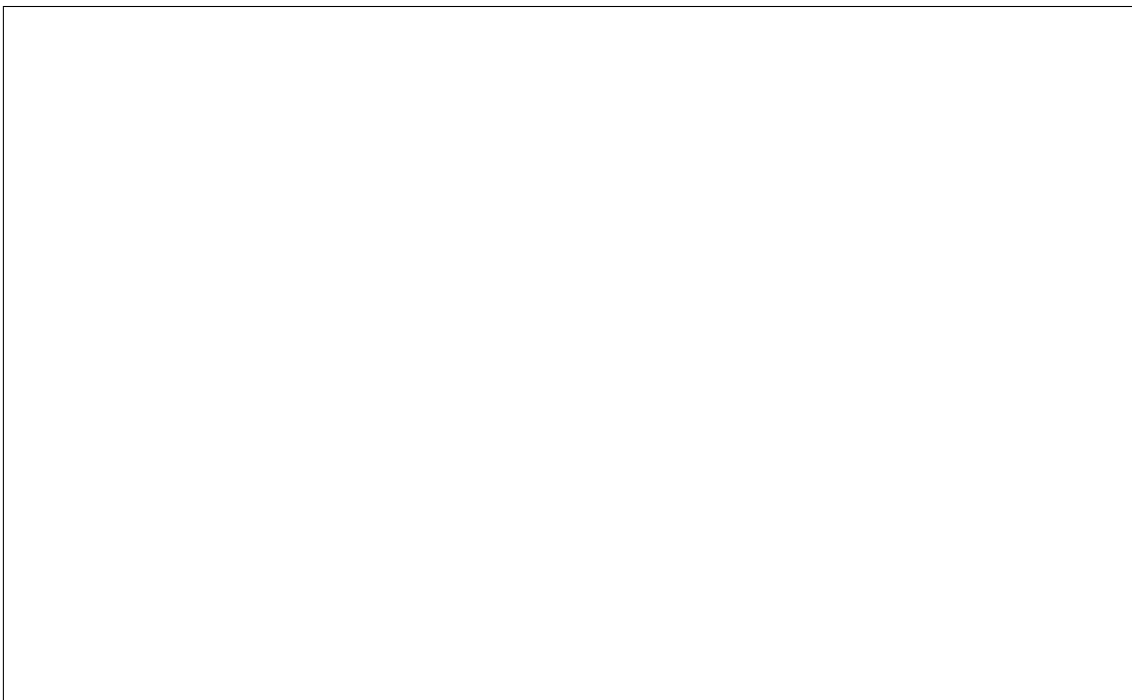
We recall that by definition, $x = y \pmod{n}$ means that n is a divisor of $x - y$.

(1) Prove that \mathcal{R} is an equivalence relation.

- (2) Assume that $n = 2$. What is the equivalence class of 0? What is the equivalence class of 1? What is the equivalence class of 2? What is the equivalence class of 3?



- (3) Going back to the general case of an arbitrary positive integer n , how many equivalence classes are there?



Problem 6 (~ 6 points.).

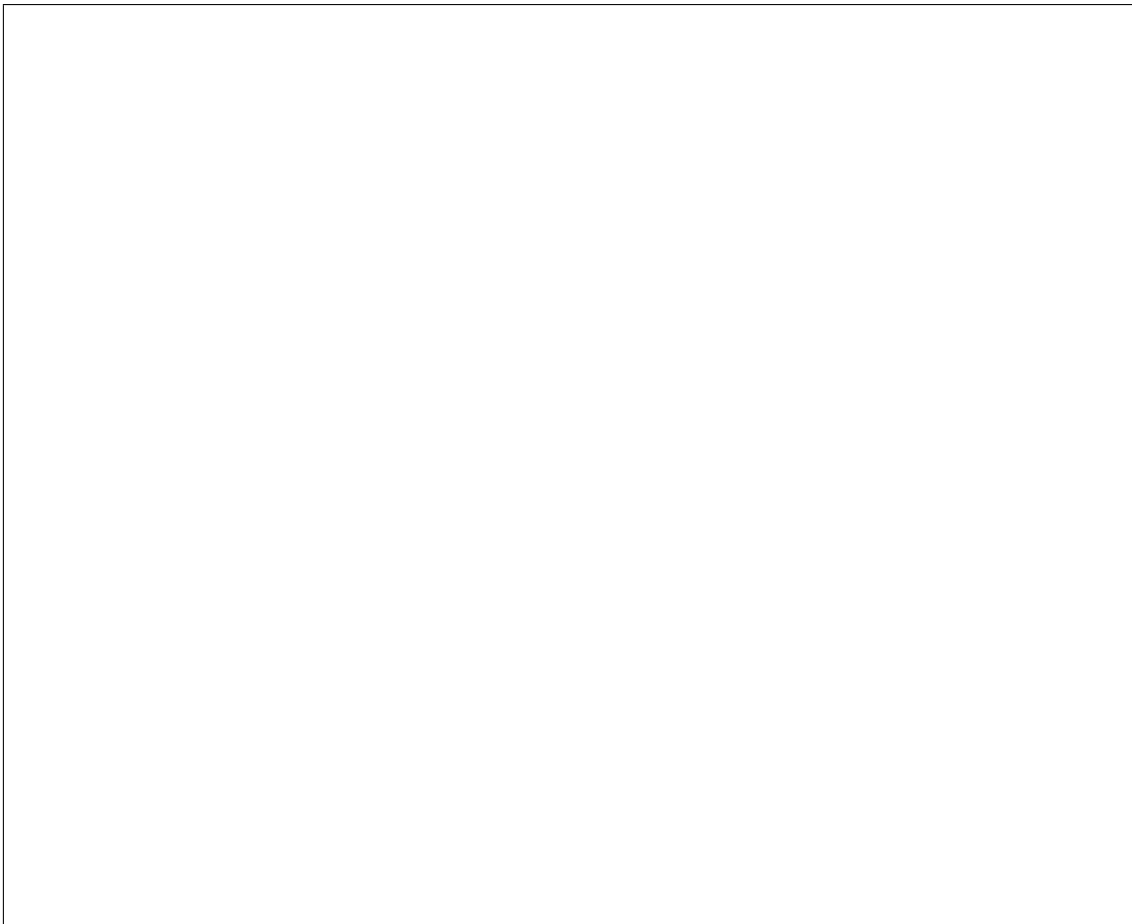
Consider the following problem: Given a finite sequence of integers $X = (X(1), X(2), \dots)$ and an integer x , find the sum of all the elements of the sequence that are greater than x .

- (1) Write an algorithm (in pseudocode) that solves the problem.

(2) Trace your algorithm for $X = (4, 3, 10, 5)$ and $x = 5$.



(3) What is the complexity of your algorithm? *Carefully explain your answer.*



Problem 7 (~ 4 points.).

True or False? *No explanations required.*

(1) $4n^3 - n^2 + 1 = O(n^3)$

(2) $4n^3 - n^2 + 1 = \Theta(n^3)$

(3) $4n^3 - n^2 + 1 = o(n^3)$

(4) $4n^3 - n^2 + 1 = O(n^4)$

(5) $4n^3 - n^2 + 1 = o(n^4)$

(6) $n^{1000} + 100n^3 = o(e^n)$

(7) $n^2 \log n = o(n^3)$

(8) $n^2 \log n = \Theta(n^2)$