

# Exam #1

Monday, October 16 2017

**Duration: 1H20**

**NAME:** \_\_\_\_\_

**Please write clearly and properly.**

<b>Problem</b>	<b>Grade</b>
<b>1</b>	
<b>2</b>	
<b>3</b>	
<b>4</b>	
<b>5</b>	
<b>Total</b>	

**Problem 1** (~ 5 points.).

Write a proof of the following theorem:

**Theorem.** *For any integer  $n \in \mathbb{Z}$ ,  $n$  is odd if and only if  $n^2$  is odd.*

**Problem 2** (~ 5 points.).

Prove or disprove the following proposition:

**Proposition.** *For any integer  $n \in \mathbb{N}$ ,  $n! + 1$  is a prime number.*

*We recall that  $n! = 1 \times \cdots \times n$ . For example,  $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$ .*




**Problem 3** (~ 5 points.).

Let  $U$  be a universe set, meaning that  $U$  contains any other set mentioned in this problem. Prove that for any sets  $A$  and  $B$ ,  $A \subseteq B$  if and only if  $\overline{B} \subseteq \overline{A}$ .

**Problem 4** (~ 5 points.).

Write a proof of the following theorem:

**Theorem.** *For any real numbers  $a$  and  $b$ , if  $a + b$  is irrational, then  $a$  is irrational or  $b$  is irrational.*



**Problem 5** (~ 5 points.).

Write a proof of the following theorem:

**Theorem.** *For any integer  $n \in \mathbb{N}$ ,  $11^n - 6$  is divisible by 5.*

*Hint: Write a proof by induction.*

