RUTGERS
UNIVERSITY | NEWARK

Math 235
Calculus III

Spring 2018

Quiz #6

Monday, March 25 2018

Duration: 25 min

NAME: _____

Please write clearly and properly.

Explain your answers appropriately.

Problem	**Grade**
1	
Total	

Problem 1 (~ 12 points.)**.**

Consider the function f of two variables defined by:

$$f(x, y) = \frac{x^2}{4} + \frac{y^2}{9} \ .$$

(1) What is the domain of definition of f?

(2) What is the equation of the graph of f? What kind of surface is this?
You may refer to the table on the last page of this quiz.

(3) Let P be the point with coordinates $(0, 3)$ in the xy-plane. Find the equation of the level curve of f through P. What is this curve? Draw a sketch of it in the xy-plane.

(4) Compute the gradient $\vec{\nabla} f(x, y)$ for any (x, y). Compute $\vec{\nabla} f(P)$ (where P is the point introduced in the previous question). Draw the vector $\vec{\nabla} f(P)$ on your previous sketch.

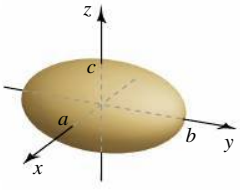
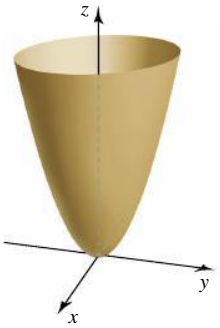
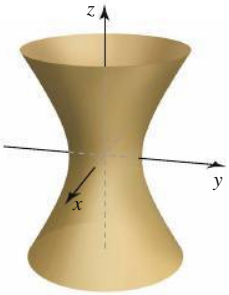
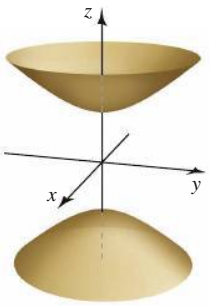
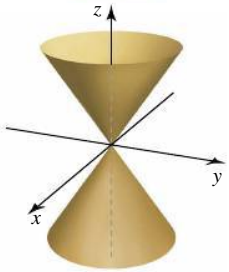
(5) Find **unit** vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ in the xy-plane such that, at the point P:

- $\vec{u}$ gives the direction of maximal rate of increase for f (steepest ascent).
- $\vec{v}$ gives the direction of maximal rate of decrease for f (steepest descent).
- $\pm\vec{w}$ gives the direction of no change for f (zero directional derivative).

(6) Compute the directional derivatives $D_{\vec{u}}f(P)$, $D_{\vec{v}}f(P)$ and $D_{\vec{w}}f(P)$.

(7) What is the tangent line to the level curve through P? Find a vector that gives the direction of this line. Check that it is orthogonal to $\vec{\nabla}f(P)$. Is this expected?

Table 12.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	All traces are ellipses.	
Elliptic paraboloid	$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Traces with $z=z_0>0$ are ellipses. Traces with $x=x_0$ or $y=y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Traces with $z=z_0$ are ellipses for all z_0. Traces with $x=x_0$ or $y=y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Traces with $z=z_0$ with $\lvert z_0\rvert>\lvert c\rvert$ are ellipses. Traces with $x=x_0$ and $y=y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	Traces with $z=z_0\neq 0$ are ellipses. Traces with $x=x_0$ or $y=y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Traces with $z=z_0\neq 0$ are hyperbolas. Traces with $x=x_0$ or $y=y_0$ are parabolas.