

## Quiz #4

Monday, February 19 2018

**Duration: 20 min**

**NAME:** \_\_\_\_\_

**Please write clearly and properly.**

**Explain your answers appropriately.**

<b>Problem</b>	<b>Grade</b>
<b>1</b>	
<b>2</b>	
<b>Total</b>	

**Problem 1** (~ 8 points.).

Consider the parametrized curve in 3-dimensional space given by the following function:

$$\begin{aligned} f: \mathbb{R} &\rightarrow \mathbb{R}^3 \\ t &\mapsto (x(t), y(t), z(t)) \end{aligned}$$

where:

$$\begin{aligned} x(t) &= \sqrt{3} \cos(t) \\ y(t) &= 2 \sin(t) \\ z(t) &= \cos(t) . \end{aligned}$$

Let  $M(t)$  denote the moving point in 3-dimensional space with coordinates  $(x(t), y(t), z(t))$ , and denote  $\vec{r}(t) = \overrightarrow{OM(t)} = (x(t), y(t), z(t))$ .

- (1) Compute the velocity  $\vec{v}(t)$ , the speed  $v(t)$ , and the unit tangent vector  $\vec{T}(t)$  for this motion.

(2) Compute the acceleration  $\vec{a}(t)$  for this motion.

(3) Show that the path lies in a sphere centered at the origin.

(4) Show that the path lies in the plane with Cartesian equation  $x - \sqrt{3}z = 0$ .

(5) Derive the nature of the curve from the two previous questions.

**Problem 2** (~ 4 points.).

Consider a moving point  $M(t)$  in 3-dimensional space whose acceleration is given by:

$$\vec{a}(t) = (6t, 0, -2) .$$

Find the velocity  $\vec{v}(t)$  and the position  $\vec{r}(t)$  for this motion, assuming the initial conditions:

$$\begin{cases} \vec{r}(0) &= (0, 0, 1) \\ \vec{v}(0) &= (-1, 2, 0) \end{cases}$$

Where is the moving point  $M(t)$  at  $t = 1$ ?