RUTGERS UNIVERSITY | NEWARK

Math 235
Calculus III

Spring 2018

Quiz #2 Solutions

Problem 1.

Recall that two vectors are orthogonal if and only if their dot product is zero.

(1) $\vec{u} \cdot \vec{v} = (-2) \times 1 + 2 \times 1 = 0$ therefore yes, $\vec{u}$ and $\vec{v}$ are orthogonal.

(2) $\vec{u} \cdot \vec{v} = 1 \times 1 + 2 \times 2 + 3 \times (-1) = 2 \neq 0$ therefore no, $\vec{u}$ and $\vec{v}$ are not orthogonal.

Problem 2.

Recall that given any two vectors $\vec{u}$ and $\vec{v}$, the cross product $\vec{w} = \vec{u} \times \vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$. Note that the other vectors orthogonal to both $\vec{u}$ and $\vec{v}$ are just the scalar multiples of $\vec{w}$.

(1) Let us compute $\vec{w} = \vec{u} \times \vec{v}$ where $\vec{u} = (0, 1, 0)$ and $\vec{v} = (0, 0, 1)$:

$$\begin{aligned} \vec{w} &= \vec{u} \times \vec{v} \\ &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \vec{k} \\ &= \vec{i} \\ &= (1, 0, 0) \end{aligned}$$

Answer: $\vec{w} = (1, 0, 0)$ is orthogonal to both $\vec{u}$ and $\vec{v}$.

Remark: A more "clever" answer to this specific question, not requiring any computations, would have been to observe that $\vec{u} = \vec{j}$ and $\vec{v} = \vec{k}$, and we know that $\vec{w} = \vec{i}$ is orthogonal to both. An even more "clever" answer, since the question did not specify that $\vec{w}$ should be non-null, is simply to take $\vec{w} = \vec{0}$ (recall that the null vector is orthogonal to all vectors!).

(2) Let us compute $\vec{w} = \vec{u} \times \vec{v}$ where $\vec{u} = (2, 1, 0)$ and $\vec{v} = (-1, 1, 0)$:

$$\begin{aligned}
\vec{w} &= \vec{u} \times \vec{v} \\
&= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} \vec{k} \\
&= 3\vec{k} \\
&= (0, 0, 3)
\end{aligned}$$

Answer: $\vec{w} = (0, 0, 3)$ is orthogonal to both $\vec{u}$ and $\vec{v}$.

Remark: A more "clever" answer to this specific question, not requiring any computations, would have been to observe that $\vec{u}$ and $\vec{v}$ are both in the xy-plane (their z-components are zero), therefore $\vec{w} = \vec{k}$ is orthogonal to both (since $\vec{k}$ is orthogonal to any vector in the xy-plane). Once again here, an even more "clever" answer, since the question did not specify that $\vec{w}$ should be non-null, is simply to take $\vec{w} = \vec{0}$ (recall that the null vector is orthogonal to all vectors!).

Problem 3.

(1) True [It is a theorem that two vectors are parallel if and only if their cross product is the null vector.]

(2) True [$\vec{k}$ = is orthogonal to any vector in the xy-plane: this is easy to check with the dot product, for instance.]

(3) True [$\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$, therefore $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.]

(4) True [This is because $\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \, \cos(\theta)$ and $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \, \|\vec{v}\| \, \sin(\theta)$, conclude using the fact that $\cos^2(\theta) + \sin^2(\theta) = 1$.]