Exam #2 Review Exercises

Problem 1.
Consider the function f of two variables defined by:

$$f(x, y) = -x^2 + y \ .$$

(1) What is the domain of definition of f?

(2) What is the equation of the graph of f? What kind of surface is the graph of f?

(3) What is the equation of the c-level curve of f?

(4) What kind of curve is the c-level curve of f? Draw several level curves of f.

(5) Consider the point A in the xy-plane with coordinates: $A(-1, -1)$. Find the equation of the level curve of f through A. Draw a sketch.

(6) Find a vector $\vec{w}_1$ giving the direction of the tangent line to the level curve through A. Add this line and this vector to your previous sketch.

(7) Compute the first order partial derivatives of f and the gradient of f.

(8) Find vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ in the xy-plane such that, at the point A:
- $\vec{u}$ gives the direction of maximal rate of increase for f (steepest ascent).
- $\vec{v}$ gives the direction of maximal rate of decrease for f (steepest descent).
- $\pm\vec{w}$ gives the direction of no change for f (zero directional derivative).

Draw a sketch.

(9) Check that $\vec{u}$ is orthogonal to $\vec{w}_1$. **Why is this expected?** Check that $\vec{w}$ is parallel to $\vec{w}_1$. **Why is this expected?**

(10) Compute the directional derivative $D_{\vec{w}} f(A)$. Is the result expected?

(11) Study the local extrema of f.

Problem 2 (Section 12.8 Exercise 31 from the textbook.)**.**

Study the extrema of the function f of two variables defined by:

$$f(x, y) = x^4 + 4x^2(y - 2) + 8(y - 1)^2 \ .$$

Hint: *In order to study the extrema of any function, you should follow the following plan:*

1. *Find the domain of definition.*
2. *Compute the gradient and find all critical points.*
3. *Study each critical point using the Second Derivative Test.*
4. *Further study each critical point where the Second Derivative Test was inconclusive, if there are any such points.*
5. *Discuss whether there are any global extrema.*

NB: *There is no general method for steps 4 and 5: one has to find concrete arguments that depend on each specific situation.*