RUTGERS UNIVERSITY | NEWARK

Math 235
Calculus III

Spring 2018

Exam #2

Wednesday, April 4 2018

Duration: 1H 50min

NAME: ________________________

Please write clearly and properly.

Explain your answers appropriately.

Calculators not allowed.

Problem	**Grade**
1	
2	
3	
Total	

Problem 1 ($\sim$ 6 points.)**.**

Consider the function f of two variables defined by:

$$f(x, y) = x^2 - y^2 .$$

(1) What is the domain of definition of f?

(2) What kind of surface is the graph of f?
Hint: Refer to the table on the last page of the exam.

(3) What is the level curve of f through the origin in the xy-plane?

(4) Does f admit a global minimum or a global maximum?

Problem 2 (~ 10 points.)**.**

Consider the function f of two variables defined by:

$$f(x, y) = 2x - y + 1 \ .$$

(1) Show that the graph of f is a plane in 3-dimensional space and give its equation. Does it go through the origin? What is a normal vector to this plane?

(2) What kind of curve is the intersection of the graph of f with a horizontal plane?

(3) Without doing any calculations, derive from your previous answer that the level curves of f are straight lines in the xy-plane.

(4) What is the equation of the c-level curve of f? Can you give a vector $\vec{w}_1$ and a vector $\vec{w}_2$ in the xy-plane such that $\vec{w}_1$ is parallel to all level curves and $\vec{w}_2$ is orthogonal to all level curves?
Hint: In the xy-plane, the straight line with equation $ax + by + d = 0$ admits $(-b, a)$ as a parallel vector and (a, b) as an orthogonal vector.

(5) Draw a sketch of the xy-plane with the c-level curves of f for a few different values of c of your choosing.

(6) Compute the gradient $\vec{\nabla} f(x, y)$. Check that it is parallel to $\vec{w}_2$. Explain why this is expected.

(7) Check that the directional derivative $D_{\vec{w}_1} f(x, y)$ is equal to zero. Explain why this is expected.

Problem 3 (~ 7 points.)**.**

Consider the function f of two variables defined by:

$$f(x, y) = 2x^3 + 6xy - 3y^2 + 2 \, .$$

(1) Find and analyze the critical points of f.
Hint: You should find two critical points: $P_1(0, 0)$ *and* $P_2(-1, -1)$.

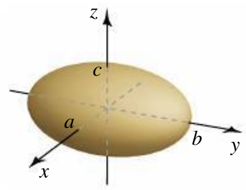
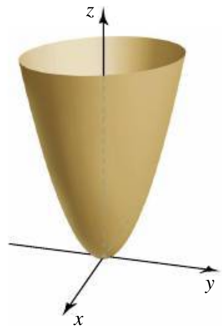
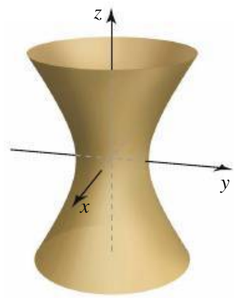
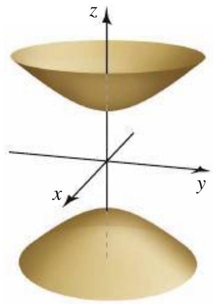
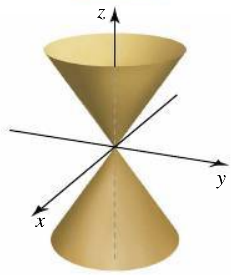
You may continue writing your solution on the next page.

You may continue writing your solution here.

(2) Compute $f(-1,-1)$ and $f(1,0)$.

(3) Does f admit a global minimum or a global maximum?

Table 12.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ with $\lvert z_0 \rvert > \lvert c \rvert$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.