RUTGERS
UNIVERSITY | NEWARK

Math 235
Calculus III

Spring 2018

Exam #1

Wednesday, February 28 2018

Duration: 1H 50min

NAME: ________________________

Please write clearly and properly.

Explain your answers appropriately.

Calculators not allowed.

Problem	Grade
1	
2	
3	
Total	

Problem 1 (~ 9 points.)**.**

Let I, A, and B be three points in 3-dimensional space given by their coordinates in the xyz-coordinate system:

$$I(1, -1, -6)$$
$$A(-2, 1, -5)$$
$$B(4, -3, -3)$$

(1) Compute the vectors $\vec{u} = \overrightarrow{IA}$ and $\vec{v} = \overrightarrow{IB}$.

(2) Find a parametric equation for the line L_1 through the points I and A. Same question for the line L_2 through the points I and B.

(3) Find a parametric equation for the line L_3 through the point I such that L_3 is perpendicular to both lines L_1 and L_2.

Hint: Start by finding a vector $\vec{w}$ which is orthogonal to both $\vec{u}$ and $\vec{v}$.

(4) Does the point $C(-1, 2, -6)$ belong to the line L_3?

(5) Check that the vector $\overrightarrow{IC}$ is orthogonal to both vectors $\overrightarrow{IA}$ and $\overrightarrow{IB}$.

Problem 2 (~ 16+2 points.)**.**

Consider the parametrized curve in 3-dimensional space given by the following function:

$$\begin{aligned} f : \mathbb{R} &\to \mathbb{R}^3 \\ t &\mapsto \big(x(t), y(t), z(t)\big) \end{aligned}$$

where:

$$\begin{aligned} x(t) &= \sin(t) \\ y(t) &= \sin(t) \\ z(t) &= \sqrt{2}\cos(t)\ . \end{aligned}$$

Let $M(t)$ denote the moving point in 3-dimensional space with coordinates $\big(x(t), y(t), z(t)\big)$, and denote $\vec{r}(t) = \overrightarrow{OM(t)} = \big(x(t), y(t), z(t)\big)$.

(1) Compute the velocity $\vec{v}(t)$, the speed $v(t)$ and the acceleration $\vec{a}(t)$ for this motion.

(2) Is $f(t)$ a parametrization by arclength? If not, find a reparametrization by arclength.

(3) Compute the unit tangent vector $\vec{T}(t)$, the principal unit normal vector $\vec{N}(t)$ and the unit binormal vector $\vec{B}(t)$.

(4) Compute the curvature $\kappa(t)$ using the formula $\kappa(t) = \frac{\|\vec{T}'(t)\|}{v(t)}$.

(5) What is the radius of curvature of this curve at any point? Make a conjecture (in other words a hypothesis) about the nature of this curve.

(6) Compute the length of the curve between $t = 0$ and $t = 2\pi$. Is your answer consistent with the conjecture you made in the previous question?

(7) Show that the path lies in a sphere centered at the origin.

(8) Show that the path lies in the plane with Cartesian equation $x - y = 0$. Does this plane goes through the origin? Does the curve go through the origin?

(9) Derive the precise nature of the curve from the two previous questions. Draw a quick sketch.

Problem 3 (~ 6 points.)**.**

An object is dropped from a height H. What is the objet's speed when it hits the ground?

You will operate under the following assumptions:

- The object is identified to a moving point in 3-dimensional space.
- The object has a mass m (in Kilograms) and is dropped from a height H (in meters).
- The object's initial velocity is null.
- The only force influencing the object's motion is the gravitational force $\vec{F} = m\vec{g}$. We recall that the gravitational field $\vec{g}$ is a constant vector pointing downwards and whose magnitude is a constant g.

Give your answer in terms of H, m, and g.

Make sure you develop a detailed answer, carefully explaining every step.

Continue writing your answer on the next page.

Continue writing your answer below.